DES

HÉLICES AÉRIENNES

Théorie générale des Propulseurs hélicoïdaux

et

Méthode de Calcul de ces Propulseurs pour l'air

PAR

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FORWARD

This text is a translation from French of a book on aerial propeller design formulated during the earliest period of aviation development. The author, Stefan Drzewiecki, was an eminent engineer who contributed to the Blade Element Theory of propeller design. According to wikipedia.org he is credited with designs for submarines, torpedo launching systems and methods of developing airplane and marine propellers from 1880 onward. His later book "Theory Générale de Hélice" (1920) was honored by the French Academy of Science as fundamental in the development of modern propellers.

I was curious about the unusual design of M. Drzewiecki's "Hélice Normale" propeller used on some pioneer aircraft. I found short descriptions of the design in French books and magazines, and a reference to this text. I could not find an online copy of the book in Google Books, Hathi Trust, Gallica or other online archives. The book is held in a few libraries around the world including the Huntington Library, in San Marino, California. Thanks to the librarians at the Huntington I was allowed an independent scholar pass to examine the book and photograph the pages.

The images from the book were converted to text using Ocropus, an open source document analysis and OCR system, and page dewarping code from Leptonica, an image processing and analysis library. After corrections to the OCR putput the text was translated to English using the Google Chrome translator feature. Since I have no knoweldge of French I have not attempted to improve on this machine translation. The PDF file was typeset in a format similar to the original using Lyx, a Latex front-end for the Mac. This project was not my original intent when I went to the library and in spite of the shortcomings of my images and the machine translation I hope someone interested in early aviation design will find the ideas and the 19th century approach to engineering described in this book interesting.

Ken Rector

AERIAL PROPELLERS

The question of propellers in general is very complex and not very well known from a theoretical point of view.

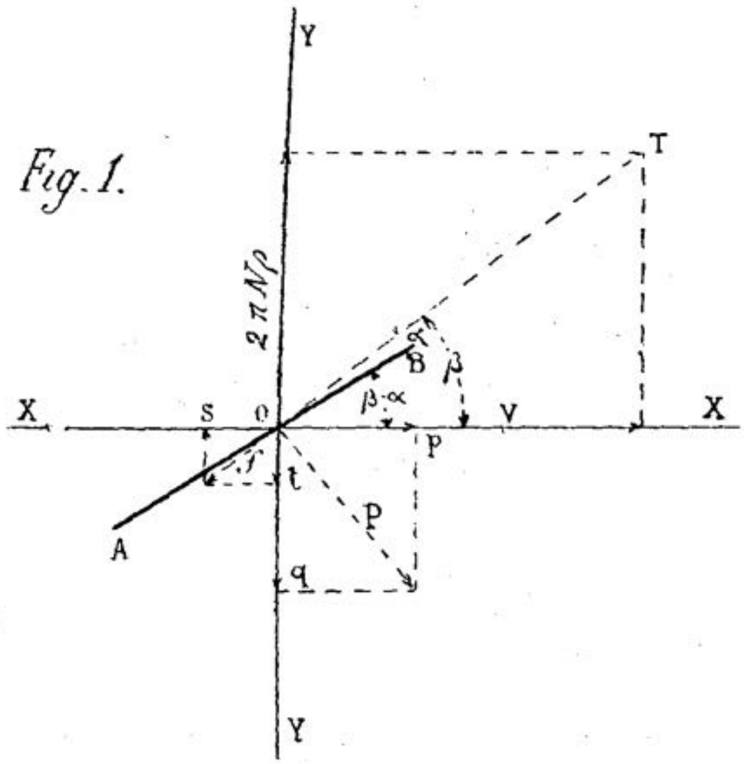
If, for marine propellers, we are, in spite of that, to achieve satisfactory uses, it is only through the accumulation of many experiments, long gropings, tests and successive modifications, often due to chance, in a word, to a practice of three quarters of a century. Also, for the calculation and sketching of the different elements of these propellers, we use, for the most part, isolated empirical methods and formulas, having no general connection between them, denoting a general conception of phenomenon. These practical processes and these formulas constitute, so to speak, traditions of workshops and personal recipes, varying with the builders and their countries. This method, however imperfect it may appear, was however, admissible when it came to what was known of marine propellers, for a boat fitted with a propeller, even a defective one, could still sail; though it was advancing slowly, consuming a great deal of coal, but the bad performance of the propeller did not prevent the boat from sailing.

It is not the same for the aerial propeller intended to propel an airplane, because, for want of a good propeller, an airplane can not be moved at all. Indeed, materials which enter into the construction of an airplane, and the laws of the resistance of these materials, constitute, for the airplane, special conditions which put it, so to speak, upon the imitates of posssble. Therefore, the use of all the elements of the apparatus, such as lift, engine and thruster, should be as high as possible, both in terms of weight and power. If only one of these elements does not meet these conditions of good use, the airplane might not leave the ground. Moreover, for the aerial propellers we do not have, as for the marine propellers, the accumulation of experiments, empirical data, coefficients, etc., provided by a long practice. That is why, for the calculation of aerial thrusters, it will be indispensable for us to form an exact mechanical conception of the operation of the propeller itself. In a word, we will have to know the general theory of the propeller and have a rigorous method to calculate all its elements without exception .

In 1892, I presented to the Maritime Technical Association a note in which I proposed a method of calculating and determining all the elements of the helical thrusters. Encouraged by the benevolent welcome my ideas have received from eminent naval engineers and distinguished scientists, I sought the verification of my theory by calculating, according to it, a large number of existing and tried propellers. I had the satisfaction of seeing that the predictions of my calculations were always in agreement with reality, and that the good propellers were precisely those which were nearest to the type indicated by the calculation. In addition, I found that certain peculiarities observed in the operation of the propellers, which could not be explained by ordinary methods, were deduced, in a simple and rational way, when considered from the point of view of my theory. Finally, this theory also gave the logical explanation of certain empirical formulas used successfully in practice. Since. a very large number of propellers have been constructed, calculated by the method I have proposed, and the results have always been consistent with the calculations. All these reasons now give me the right to consider the principles of this theory as perfectly correct. We will therefore try to apply them also to the calculation of aerial propellers .

Consider a point O (fig 1) located on a rigid radius, fixed perpendicular to an axis XX rotating jointly with he, at a uniform speed of N turns per second; suppose at the same time that the axis XX itself is animated by a movement translation in the direction of its length, with a speed uniform V.

Fig. 1.



Because of the rotation around the axis XX, the point O will be animated, in the direction OY, at a peripheral speed which will be expressed by $2\pi N\rho, \rho$ being the distance from the point O to the axis of rotation XX. On the other hand, because of the longitudinal advancement of the axis of rotation itself, the point O will also have a velocity V in the direction parallel to the axis XX. The actual velocity of the point O will be the resultant of the two velocities $2\pi N\rho$ and V. It will therefore be represented, in magnitude and in direction, by OT, a diagonal of the rectangle, whose sides are respectively $2\pi N\rho$ and V. This diagonal will also represent the tangent to the real trajectory followed by the point O; the helical line resulting from the winding of the diagonal OT on the cylindrical surface, whose axis would be XX, and radius ρ, The pitch of this helical line would be $\frac{V}{N}$, which also represents the advance per revolution. Calling β the angle this tangent OT makes with the generator of the cylinder projected at XX, we have:

$$tang\beta = \frac{2\pi N\rho}{V} \text{ and } 2\pi N\rho = V.tang\beta$$

If the movement of the point O takes place in a fluid at rest, the velocity of the point O with respect to the fluid will be directed along OT, or, what is the same, the fluid threads will meet the point O in the direction TO.

Fix, at the point O on the radius ρ, a plane element passing through this radius, and projecting onto the XY plane following AB. Moreover, let us orient this element so that it makes with the direction OT an angle α, within the angle β, so that the angle of the plane element with the X axis is (β-α).

When the element AB will be dragged into the motion of the point O, along the helical path OT all the points of the element AB will meet the fluid threads in directions parallel to OT and consequently under an incidence α.

Aerodynamics teaches us that under these conditions the plane element will experience, from a fluid encountered, a certain resistance, the direction which is, for the moment, unknown. This resistance will be proportional to the extent of the surface, the square of the velocity and very approximately to the sine of the angle of incidence. It will be expressed by the formula $R = K.S.W^2.sin\alpha$, where K is an empirical coefficient, and W is the velocity of the element in the fluid at rest, This resistance can always be represented by its two components; one P, directed perpendicular to the tangent to the trajectory, and which we shall call the useful thrust, and the other f, directed along OT, but in the opposite direction, and which will represent the harmful resistance. Whatever the direction of resistance and the numerical values of these two forces P and f, they can always be connected by a relation such that $f = \mu, P$ where μ is a constant coefficient independent of the velocity, since f and P are the components of the same resistance, and vary both at the same time, and in the same way, proportionally to the square of the velocity.

The two components being in a plane parallel to the XY plane, their projections on the Z axis will be zero, while their respective projections on the X and Y axes will be:
for the X:

$$+Op \text{ and } -Os$$

and for the Y:

$$-Oq \text{ and } -Ot$$

Expressing these projections as functions of P, μ and β,
we have:

$$\begin{aligned} Op &= Psin\beta \\ Os &= fcos\beta = \mu Pcos\beta \\ Oq &= Pcos\beta \\ Ot &= fsin\beta = \mu Psin\beta \end{aligned}$$

The algebraic sum of these projections on the X axis will be:

$$Op - Os = P(sin\beta - \mu cos\beta)$$

and on the Y axis:

$$-(Oq + Ot) = -P(cos\beta + \mu sin\beta)$$

If we consider the system as an elementary thruster powered by a motor which develops a motor torque F, intended to balance the running resistance $-R$ of the whole system, in the direction of the axis XX, we can pose:

$$-R + P(sin\beta - \mu cos\beta) = 0$$

and as this resistance R, occurs at the speed V, we can deduce the value of the useful power, or the useful work per second, by multiplying the resistance by the speed.

$$\zeta_u = RV = PV(sin\beta - \mu cos\beta)$$

Similarly, the following component OY will constitute at the distance ρ of the axis of rotation, a torque that will balance the motor torque F, we will have:

$$F\rho - P\rho(cos\beta + \mu sin\beta) = 0$$

and multiplying by 2πN, which is the tangential velocity of the engine torque, we will have the motive power, or the motor work per second:

$$\zeta_m = 2\pi N\rho F = 2\pi N\rho P(cos\beta + \mu sin\beta),$$

but, as we have seen:

$$2\pi N\rho = V tang\beta$$

we have:

$$\zeta_m = PV(cos\beta + \mu sin\beta)tang\beta$$

By dividing the expression of the useful power ζ_u, by that of the motive power ζ_m, we will have the utilization coefficient K, of this elementary thruster. It will be:

$$K = \frac{\zeta_u}{\zeta_m} = \frac{sin\beta - \mu cos\beta}{(cos\beta + \mu sin\beta)tang\beta}$$

dividing the numerator and the denominator by $cos\beta$, we have:

$$K = \frac{tang\beta - \mu}{(1 + \mu tang\beta)tang\beta}$$

which determines the value of the use of the different elements of a thruster, for different values of tang β.

By giving μ the form

$$\mu = tang(arctang\mu)$$

the expression of K becomes, after simplification

********* missing lines ************

If the friction and the resistance due to the thickness of the element AB did not exist and the ratio $\mu = \frac{f}{P}$ was reduced to $tang\alpha$, the use K would be expressed by:

$$K = \frac{tang(\beta - \alpha)}{tang\beta}$$

This shows that this use would be larger as the angle α would be smaller, and that K would become equal to unity for $\alpha = 0$.

Let us take the general expression of K:

$$K = \frac{tang\beta - \mu}{(1 + \mu tang\beta)tang\beta}$$

we see that, for values of tang β less than μ, the coefficient K is negative; that it is zero for $tang\beta = \mu$, and that, for increasing values of $tang\beta$, it increases, passes by a maximum and then decreases until 0, when $tang\beta$ tends to infinity.

To determine the value of $tang\beta$, which makes maximum the value of K, equal to zero the first derivative of the expression; after simplification we find:

$$tang^2\beta - 2\mu tang\beta - 1 = 0$$

Solving this equation and taking only the positive value of $tang\beta$, we find that the value of $tang\beta_M$, which makes K maximum, is:

$$tang\beta_M = \mu + \sqrt{\mu^2 + 1}$$

Substituting this value in the expression of K, we find for the value maximum of K_M:

$$K_M = \frac{1}{\left(\mu + \sqrt{\mu^2 + 1}\right)^2}$$

The denominator is precisely the square of $tang\beta_M$, so:

$$K_M = \frac{1}{tang\beta_M}$$

As the coefficient K approaches unity, the value of $tang^2\beta_M$ should be close to

1, and therefore also $tang\beta_M$ itself; this shows that the maximum efficiency of the thruster will correspond to that part of the neighboring wing $tang\beta = 1$, which gives $\beta = 45^\circ$. This part is near the base of the wing. This proves that it is not the end of the wing, as is generally believed, but its base, which is the part of the thruster giving the best use.

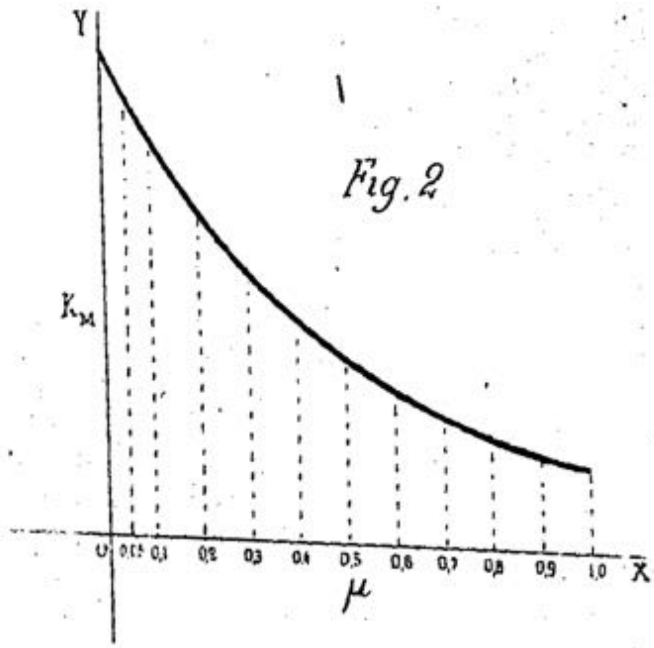


Fig. 2

Let us draw the curve of the values of K_M, by varying μ from 0 to 1, by increases of 0.1 (fig. 2). The ordinates of this curve are entered in the table below (Table A) below the corresponding values of μ and the corresponding values α. We see that K_M decreases very rapidly with the increase of μ and that it is, there-fore, extremely important for the thrusters, as μ, and hence also α, are as small as possible.

TABLEAU A

μ =	0,05	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0
α =	1°50′	4°40′	10°20′	13°45′	20°55′	25°45′	30°12′	34°18′	38°4′	41°25′	44°30′
K_M =	0,905	0,819	0,692	0,554	0,459	0,382	0,321	0,271	0,231	0,198	0,172

The value of μ consists of two parts: the first, and the most important, depends on the incidence, since μ is proportional to $tang\alpha$; the second part of the resistance is due to the friction of the fluid on the surface of the propeller element and to the section of this element. This second part of the resistance is very small compared to the first; it depends on the greater or lesser thickness of the wing and the state of its surface, so it is independent of α and constant for the same wing at all angles. The values of these two parts of μ for increasing values from $\mu = 0.05$ to $\mu = 1$, adopting for μ the form $\mu = tang\alpha + 0.018$. This value, which we have attributed to the part of the resistance due to the friction and thickness of the element under consideration, is evidently a little arbitrary, but we have no positive data to deduce it. It is only in an aerodynamic test laboratory that it would be possible to determine it accurately. In any case the error must not be considerable, and the values of the angle of incidence α entered in the last line of Table A must not differ significantly from reality.

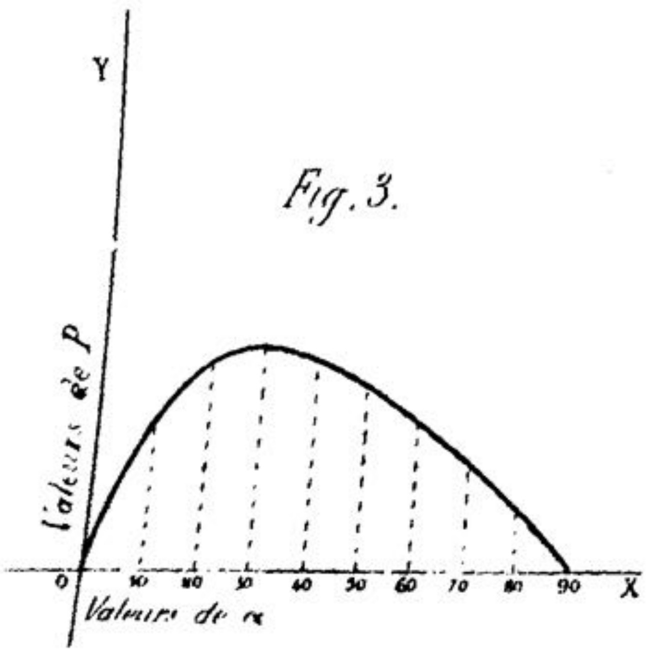
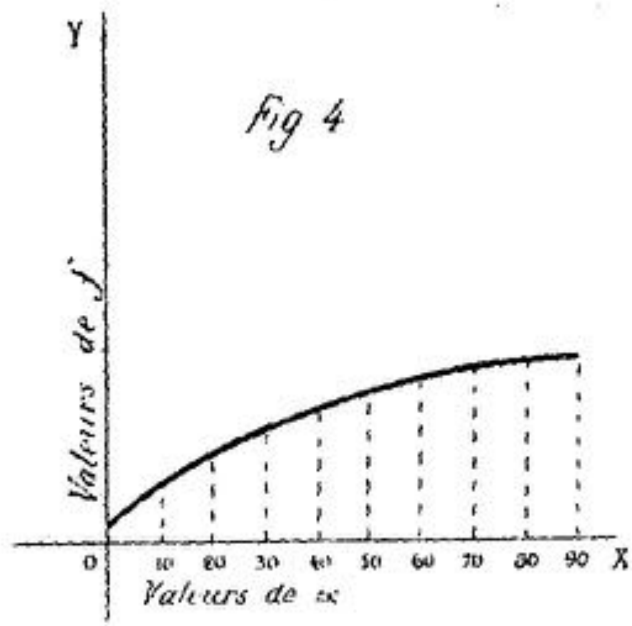


Fig. 3.

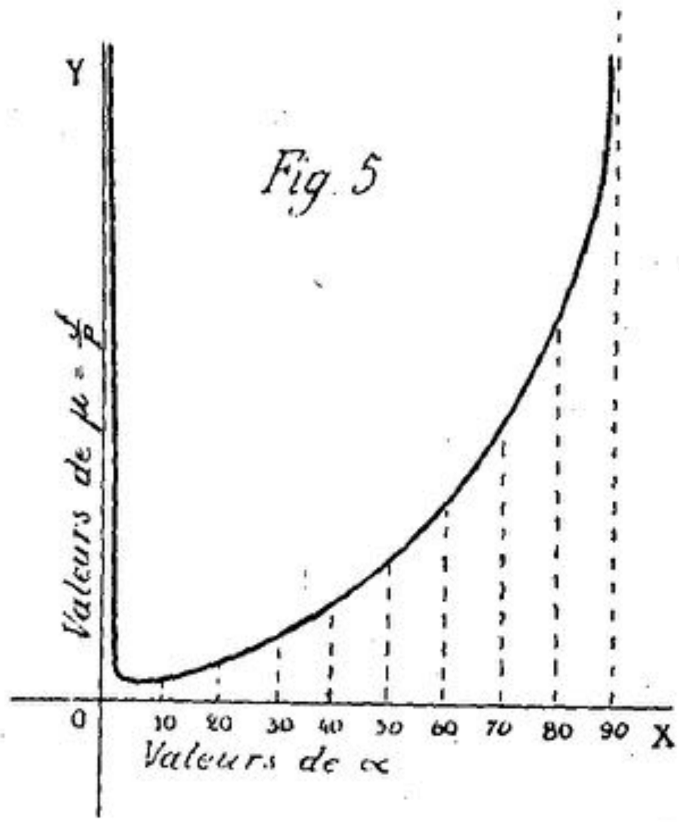
At inspection of the table we see that to obtain, for a helical propeller, an advantageous yield, it is necessary that its elements attack the air at the lowest possible incidence. We will see however that there is a limit for α, below which it is no longer advantageous to descend.

To demonstrate this, graphically represent the increasing values of P and f taking as the abscissa the increasing values of the incidences from $\alpha = 0^\circ$ to $\alpha = 90^\circ$.

The curve, whose ordinates represent the values of P (fig. 3), corresponding to those of α, starts at zero, since for $\alpha = 0^\circ$ the value of P is zero, then it goes up to a maximum, which corresponds to approximately 27° or 30°, then goes down gradually to 0, when α reaches 90°; at this moment the force P, perpendicular to the trajectory, vanishes.



Similarly draw (fig 4) the curve of the values of f, for increasing values of α we see that for $\alpha = 0^\circ$, f is represented by a certain value, which depends on the friction and the thickness of the moving element. This value is very small, but it is measurable. By increasing the values of α from 0° to 90°, those of f also increase to a maximum corresponding to $\alpha = 90^\circ$. Draw a third curve (fig. 5). always taking for the abscissae the increasing values of α from 0° to 90°, and for ordinates the corresponding values of $\mu = \frac{f}{P}$, which represents the ratio of the ordinates of the curve f (fig. 4) to the corresponding ordinates of the curve P (fig. 3). We see that for $\alpha = 0^\circ$, μ is infinite since it is the quotient of the value of f_0, a low but positive value, by the value of P_0 which is zero. The μ curve is therefore asymptotic to the Y axis; it drops rapidly to a minimum value, which corresponds to a value of μ close to 2°, and then returns to become asymptotic again at an ordinate passing through $\alpha = 90^\circ$, since at this moment $\mu = \frac{f_{90}}{0}$.



This minimum value for α, neighboring 2° was determined by us in an earlier study from 1887, Therefore, based on the formulas of Duchemin, we calculate the different impacts under which a square meter of the load-bearing surface could hold a given weight in the air, advancing horizontally with a determined speed; then we drew up a table, of eight columns, in which each column corresponded to a bearing weight of 1,2,3 .... 8 kil. The horizontal lines of the table corresponded to the incidences required to carry the weights in question, at speeds increased from 5, 10, 15 .... to 30 meters per second; these incidences were evidently decreasing in each of the eight columns. We have calculated, in the same manner and under the same conditions, a second table in which were ranked the power required to advance the square meter, successively loaded increasing weights of 1 to 8 kil., and meeting the air under the decreasing incidences, determined above, and with increasing velocities of 5 to 30 meters. The two tables were absolutely similar, so that each value of one table corresponded to the similar value of the other. In examining the table of motor powers, it was found that, in each column, the power passed through a minimum, and it was found that this minimum corresponded, in the first table, for all the speeds and for all the weight carried, always at the same incidence very close to 2 °. By interpolating, we found more exactly $\alpha = 1^\circ 50'$. It is at this angle that we gave the name of *optimal incidence*. The numerical value of the optimal incidence, thus found, obviously depends on the coefficients employed in the different formulas adopted, and its exact value can be rigorously determined only by direct tests in an aerodynamic laboratory. For the moment, in our later calculations, for want of more precise data, we will adopt this value of $\alpha = 1^\circ 50'$, which seems, moreover, very close to what it really must be. For water, we adopted $\alpha = 3^\circ$, which proved quite accurately in the calculation of the marine propellers. The very strictness in the appreciation of the optimal incidence is also of little importance, a slight difference in one direction or the other will not influence in a very significant way the performance of the propeller, provided that the gap is not too big.

By the foregoing, we see that, in the aerial propellers, it is advantageous to dispose the elements constituting the surface of the propeller, so that they meet the fluid threads under a very small and constant incidence, the optimal incidence $\alpha = 1^\circ 50'$, and that, under these conditions, the use of the thruster is maximum.

Let us see now how we are going to arrange these elements along the radius ρ, and if it is not necessary to take on this radius a determined length between certain limits. For that, let us take the expression

$$K = \frac{tang\beta - \mu}{(1 + \mu tang\beta) tang\beta}$$

Let us draw a first curve taking the increasing values of $tang\beta$ for abscissae and assigning to μ a given value $\mu = 0.05$; the ordinates of this curve will be the corresponding values of K (fig.6).

We see that the curve is asymptotic to the axis of Y in its negative part, since $K = -\infty$ for $tang\beta = 0$; the curve rises quickly to cut the X axis; $K = 0$ for $tang\beta = \mu = 0.05$. Then the curve rises rapidly to pass through a corresponding maximum $K_M = 0.905$, as we have seen, to $tang\beta_M = \mu + \sqrt{\mu^2 + 1} = 1.051$, after which, it gradually decreases, but slowly, to become asymptotic to the X axis for $tang\beta = 0$.

Tableau B

μ	tang β =	0,5	1	2	3	4	5	Moyenne de K.
0,05	K =	0,878	0,905	0,886	0,853	0,823	0,792	0,855
0,1	K =	0,762	0,818	0,792	0,744	0,696	0,653	0,744
0,2	K =	0,545	0,667	0,643	0,583	0,528	0,480	0,579
0,3	K =		0,538	0,530	0,474	0,421	0,376	0,471
0,4	K =		0,429	0,444	0,394	0,346	0,306	0,388
0,5	K =		0,333	0,375	0,333	0,292	0,257	0,324

In the table above (Table B), we have entered in the first horizontal row the increasing values of $tang\beta$ from $tang\beta = 0.5$ to tang $\beta_1 = 5$. Below we have arranged the corresponding values of K, giving μ a value $\mu = 0.05$. At the end of this row, we

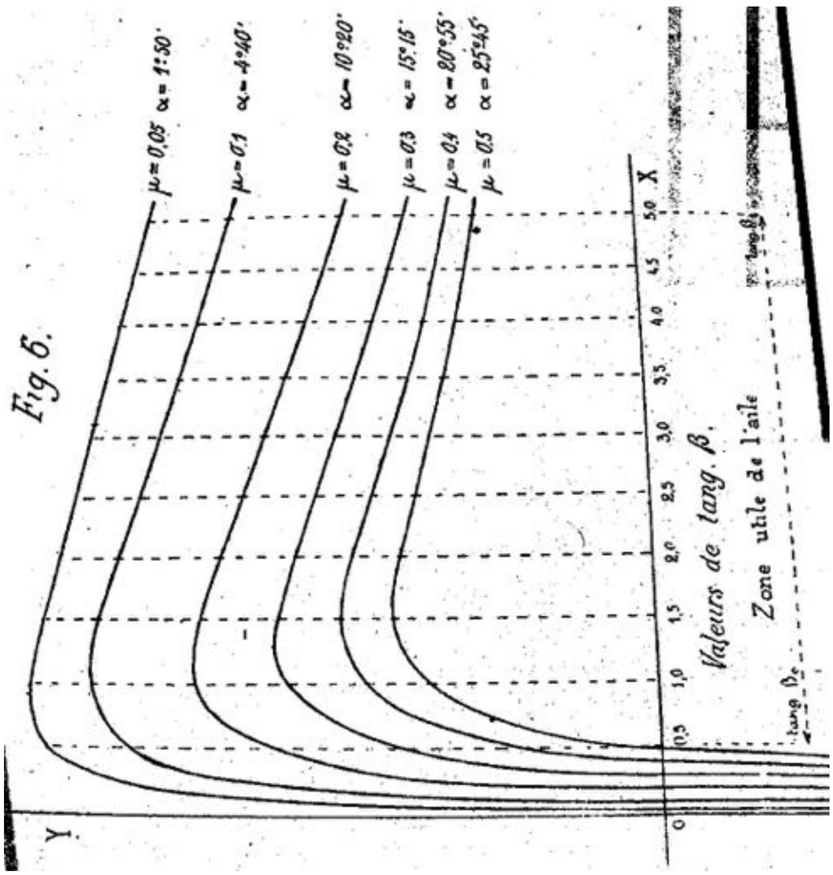


Fig. 6.

indicated the average values of K of said line; this average is $K = 0.855$. This figure shows the average return that could be obtained with a helical propeller, whose elements would attack the air under the optimum incidence and that would be placed along the radius ρ from $\rho_0 = 0.5$, until $\rho_1 = 5$.

We see that we do not have an interest in using the part of the thruster which lies between the axis of rotation and the value $\rho = 0.05$, because the yields there are very low, and even negative; also, the propulsive surface of the wing must start from a radius corresponding to a value $tang\beta$ equal to 0.5. Likewise, the length of the wing must not exceed a value corresponding to tang $\beta = 5$, since beyond this length, the values of K decrease appreciably, We will call r_0, andr_1 the radii that correspond to:

$$tang\beta_0 = 0.5 \text{ and } tang\beta_1 = 5,$$

so that we can write:

$$r_0 = \frac{V}{2\pi N} tang\beta_0 = 0.5 \frac{V}{2\pi N},$$

and

$$r_1 = \frac{V}{2\pi N} tang\beta_1 = 5 \frac{V}{2\pi N} \ .$$

The expression $\frac{V}{N}$ is, what is called, the *advance per turn* of the thruster and we will designate it by $A = \frac{V}{N}$; we will call *Module* the advance A, divided by 2π, and we will designate it by:

$$M = \frac{V}{2\pi N} = \frac{A}{2\pi} \ .$$

This expression will play an important role in the following, and is why we determine it now. By replacing in the expressions of r_0, and r_1, which define the radius of the hub and that of the wing of the thruster, $\frac{V}{2\pi N}$. by the module M, we will have:

$$r_0 = 0.5M \text{ and } r_1 = 5M \ .$$

This shows that, for an average efficiency aircraft propeller wing, $K = 0.855$, which we shall call the *normal wing*, the beginning of the wing will correspond to a length of radius equal to half of the module, and the total length of the wing will be equal to 5 modules.

By examining the figures in the table above, one wonders why it would not be enough to use only the part of the wing between the limits $tang\beta_0 = 0.5$ and tang $\beta_1 = 3$ or 4 for example, for which the average yields would be higher; these are indeed the limits which have been allowed for the normal wings of the marine propellers, but for the aerial propellers these limits would be too limited, for it is essential for the aerial thrusters to have a considerable propulsive surface; moreover, as the shape of the wings will have to be also narrower, than that of the marine propellers, by stopping the length of radius at 3 modules, or even 4, it would be necessary to give the propellers too many wings to make the necessary propulsive surface. That is why we were led to adopt for the normal wing in the air $r_1 = 5$ M.

In practice, it may happen that this assumed length for the normal wing is still insufficient, for example for propellers rotating very quickly, and for which the advance per turn is low, as a result of which the module M will also be very small; In order to avoid the use of too many wings, it will be necessary to increase the value of r_1 to 6, 7 and even 8 moduli, while on the contrary, rare will be the case where the opportunity to go below $r_1 = 5$ M.

In all that follows we will always call the *normal* wing, a propulsor wing whose elements attack the air under the optimum incidence $\alpha = 1^\circ 5o'$ and whose wing lengths are determined by the relations:

$$r_0 = 0.5M \text{ and } r_1 = 5M \ .$$

In the course of the present study we shall very frequently have occasion to deal with the normal wing, which has been established on the basis of a certain convention, as we have just seen. This convention has the immense advantage of determining immediately and completely all the elements of the normal wing, by means of the module only, and moreover, the conditions adopted in this convention are very similar to those generally encountered in the practice of aerial propellers, so that, in most cases, the standard of motion may be applied as it is. without any modification.

We have just seen that, for the normal wing, the average yield can reach the value $K = 0.0855$ when $\mu = 0.05$, but that it decreases rapidly with the increase of μ.

To show in a striking way the rapid fall of the average coefficient of utillisation with the increase of μ, and consequently of the incidence α, we have plotted the curves of these uses, and taking values of μ increasing by 0. 1, 0.2, 0.3, 0.4, and 0.5. We have grouped these curves in the same figure as the first one, which corresponds to $\mu = 0.05$ (fig. 6). In addition, the ordinates of these curves have been entered in Table B next to the corresponding value of μ; in the last column are ranked the average values of the use for each case.

One can easily realize the rapid fall of this average coefficient with the increase of μ; the average coefficient which is 0.855 when $\mu = 0.05$, falls to 0.323 for $\mu = 0.5$. As it is the value of $tang\alpha$ which mainly influences that of μ, there is a very great interest in arranging the wing so that the incidence is as small as possible. It is never to be feared that this incidence is too small, even if it is less than 1 ° 50' because in this case, the propeller resistant torque becoming lower than the expected engine torque, the engine will race; it will follow that the number of turns of the propeller, being larger than normal, the advance per turn will decrease, which will result in increasing the incidence, until it reaches its value. The most useful is then that the engine torque will balance the resistant torque. For a well calculated propeller. this equilibrium will be done automatically for the optimal incidence.

In the foregoing, we have learned to determine the angle of attack and the length of the wing needed to obtain the maximum efficiency of the thruster; we saw that for this, it was necessary to arrange along the radius ρ, between certain limits *****missing lines here ***** resulting from the juxtaposition of the elements AB along the radius ρ, for increasing values of $tang\beta$, is a helicoidal surface whose pitch is:

$$H = \frac{2\pi\rho}{tang(\beta - \alpha)},$$

but as:

$$tang(\beta - \alpha) = \frac{tang\beta - tang\alpha}{1 + tang\beta tang\alpha},$$

we have:

$$H = \frac{2\pi\rho(1 + tang\beta tang\alpha)}{tang\beta - tang\alpha},$$

replacing $2\pi\rho$ by its value:

$$\frac{V tang\beta}{N},$$

we get

$$H = \frac{V(1 + tang\beta tang\alpha)tang\beta}{N(tang\beta - tang\alpha)} \;.$$

Let us note in passing that, in this expression, the function which multiplies the advance $\frac{V}{N}$, is precisely the opposite of that that we had found for K, and in which μ would be replaced by $tang\alpha$; this shows that, if the friction did not exist and that μ is reduced to $tang\alpha$, we would have:

$$H = \frac{V}{N} . \frac{1}{K}$$

or:

$$K = \frac{A}{H}$$

which shows that the performance would be measured by the ratio of advance per pitch.

In examining the pitch value:

$$H = \frac{V(1 - tang\beta tang\alpha)tang\beta}{N(tang\beta - tang\alpha)},$$

we see that this pitch is a variable pitch; that he is infinite for $tang\beta = tang\alpha$, which occurs when $\beta = \alpha$, in the neighborhood of the axis of rotation. As $tang\beta$ increases, the value of H decreases, it goes through a minimum that is determined when the first derivative of the function equals zero; we find so after simplification:

$$H_{\mathrm{m}} = \frac{V}{N}\left(tang\alpha + \sqrt{tang^2\alpha + 1}\right)^2$$

corresponding to a value of:

$$tang\beta_m = tang\alpha + \sqrt{tang^2\alpha + 1}$$

For $tang\beta = \infty$, the pitch becomes infinite again.

Here again the similarity of expression is striking between the minimum of pitch H_m, and the inverse of the maximum yield K_M, which we have deduced above, In these two expressions it is the same value of $tang\beta$ that makes, one the minimum, the other maximum, with the only difference that $tang\alpha$ is replaced by μ. The value of the minimum pitch can be expressed as a function of $tang\beta_m$ that makes it minimum:

$$H_m = \frac{V}{N}.tang^2\beta_m,$$

but, as on the other hand:

$$H_m = \frac{V}{N}.\frac{tang\beta_m}{tang\left(\beta_m - \alpha\right)},$$

so:

$$tang^2\beta_m = \frac{tang\beta_m}{tang\left(\beta_m - \alpha\right)},$$

or:

$$tang\beta_m.tang\left(\beta_m - \alpha\right) = 1.$$

We can deduce that $tang\beta_m$ is greater than unity exactly the same amount as $(tang\beta_m - \alpha)$ is less than 1, therefore:

$$\beta_m = 45^\circ + \frac{\alpha}{2} \text{ and } \beta_m - \alpha = 45^\circ - \frac{\alpha}{2}$$

and since $\alpha = 1^\circ 50$, we will have β_M =45 ° 55' and β_M- α= 44 ° 5'.

Replacing both $tang\beta m_M$ and tang $(\beta_m - \alpha)$ by their numeric values, we find;

$$H_m = \frac{V}{N}\frac{1.032}{.994} = 1.038\ \frac{V}{N} \text{ and } tang\beta_m = 1.032$$

We see that the minimum pitch corresponds to a value of $tang\beta$ very close to the unit, it is therefore in the part of the wing which is inclined on the axis of an angle a little higher at 45 ° , more exactly at 45 ° 55", which corresponds to a length of radius ρ slightly greater than module; it is also the place of the maximum yield.

For the drawing of marine propellers, a very convenient method is generaly used which consists in carrying on the X axis, a length equal to the pitch divided by 2π and to lead, from the point thus determined, a series of straight lines which come to meet the radius, at different heights: for example, a quarter, a half, or three quarters of its length, the inclinations of these lines determine the inclination of the wing at these different heights, that is to say, determine the pitch of the helix corresponding to these different points of the wing.

This is easily explained by the similarity of the triangles thus obtained, with those which would have, on the one hand, the pitch H and on the other, the development of the circles whose radii would be respectively $\frac{1}{4}\rho$, $\frac{1}{2}\rho$ $\frac{3}{4}\rho$ and which would therefore be $\frac{1}{2}\pi\rho$,$\pi\rho$, and $\frac{3}{2}\pi\rho$.

For aerial propellers, we will adopt an analogous plot, only we will choose the points on radius ρ, at distances from the axis that would be multiples of the module. We have seen that the beginning of the wing must correspond to a value of the radius equal to 0.5M, that the maximum of yield as well as the minimum of pitch correspond to $\rho = M$, and that the normal wing had a length of 5 M. We will subdivide the wing (fig. 7) in equal parts corresponding to increasing values of the module:

0.5 M, 1 M, 2 M, ... 5 M.

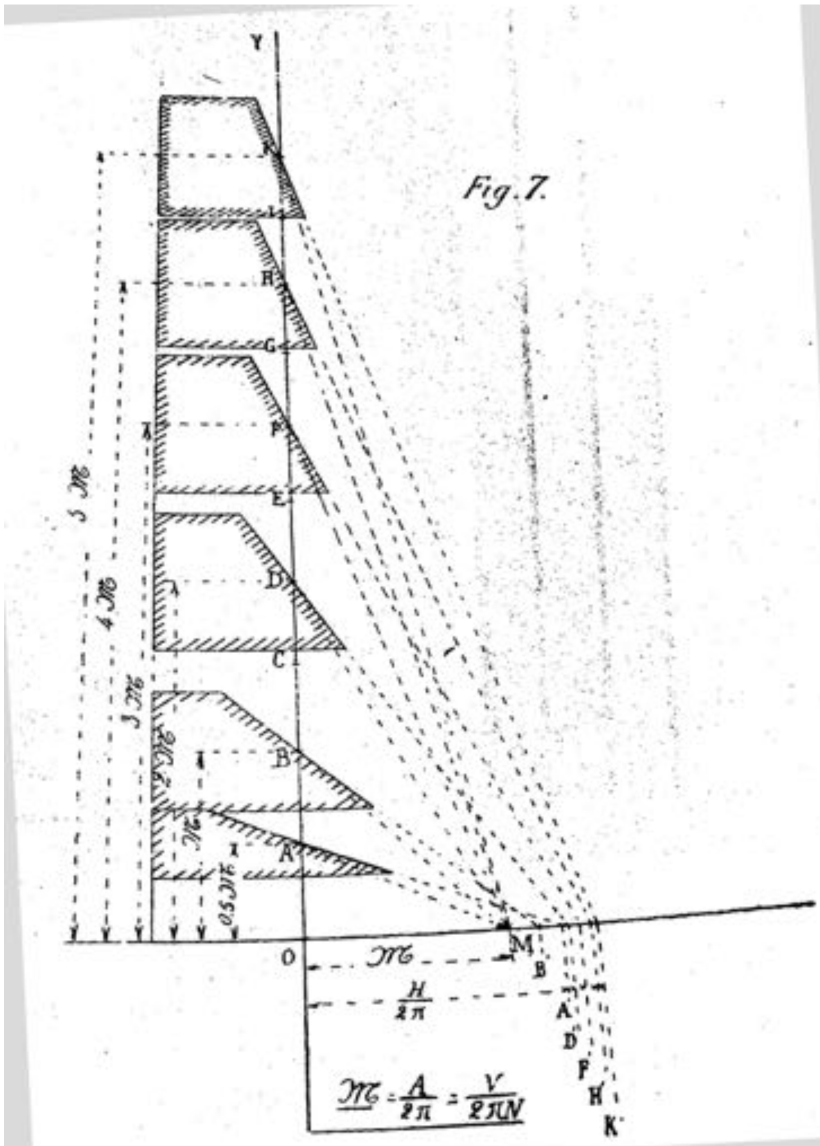


Fig. 7.
Y
O
M
$\frac{H}{2\pi}$
A
B
C
D
E
F
G
H
K
A
D
F
H
K
5 𝔐
4 𝔐
3 𝔐
2 𝔐
𝔐
0.5 𝔐
$\mathfrak{M} = \frac{A}{2\pi} = \frac{V}{2\pi N}$

The points A, B, C, D, E, ... K, thus determined, we will join them at point M, taken on the axis of rotation, at the distance from the origin of the wing equal to module; the directions MA, MB, MC, ..., MK, will represent the tangents to the helical trajectories that describe in the air the different points A, B, C, ... K. At these different points, the surface of the wing will, with these directions, an angle of incidence $\alpha = 1^\circ 50'$, and the different sections of the wing, at these points, be represented by rectilinear elements which will be inclined on the lines MA, MB, ... MK, of the angle α. These rectilinear extended elements come will have to cut the OX axis at different points, whose distance to the origin O, will be for each point, equal to pitch divided by 2π, For a constant pitch helix, all these points merge into one, since the pitch is the same for all the points of the radius, but then the incidences are variable and diminish towards the end of the wing; while for the wing with constant incidence, it is, on the contrary, the pitch which increases towards the end of the wing, All the values of the pitches, divided by 2π, that we adopted for our wing, at constant angle of incidence, can be calculated once and for all, because they always correspond to the same subdivisions of the radius: moreover, they can be expressed according to the module. These values are obtained in the following way, We saw that:

$$H = \frac{V}{N tang(\beta - \alpha} \text{ and } \frac{H}{2\pi} = \frac{V tang\beta}{2\pi N tang(\beta - \alpha)},$$

but as:

$$\frac{v}{2\pi N} = M,$$

we will have:

$$\frac{H}{2\pi} = \frac{tang\beta}{tang(\beta - \alpha)} M$$

We can calculate, once and for all, the values of $tang(\beta - \alpha)$ corresponding to increasing values of $tang\beta$, then $tang\beta = 0.5$, 1, 2, 3, ... 5, and divide the second by the first. In the appendix table (Table C), these values have been calculated up to $tang\beta_1 = 8$, because in practice, the length of wings must exceed the value of $tang\beta_1 = 5$, which we have adopted for normal wing, and reach the value corresponding to $tang\beta_1 = 8$.

When examining the table, we see that for the increasing values of $tang\beta$, the values of $\frac{H}{2\pi}$ begin by decreasing, since $\frac{H}{2\pi} = 1.085M$, for $tang\beta_0 = 0.5$, up to a minimum value $1.066M$ for $tang\beta = 1$, then increase gradually with the increase of $tang\beta$. We also see that $\frac{H}{2\pi}$ is expressed in terms of the module. On the other hand, the module M represents itself the advance A, divided by 2π, so we have, for each value of $tang\beta$, to take the corresponding value H as a function of the advance, and successively write $H = 1.085$ A, $H = 1.066$ A, and so on; we see here that the pitch has not exceeded the advance of a quantity which is respectively 8.5% and 6.6%, of this advance; it is this excess of the pitch on the advance which constitutes the *retreat* of the wing, for the point of the wing considered. For wings with constant incidence, and therefore with variable pitch, this retreat is variable, it begins by decreasing the hub to the length of the wing which is equal to the module, then it continuously increases until the end of the wing.

On the contrary, for constant-pitch and variable angle of attack propellers, the retreat is constant, for all the points of the wing, and it is the angle of attack which decreases; it diminishes even so much that for a wing a little long, exceeding for example 3 modules, this angle becomes excessively small and the efficiency of the helix decreases considerably; for if the wing is built so as to have a proper retreat, the part of the wing adjacent to the hub will have a too high angle of attack and the neighboring part of the extremity an angle of attack too low, and in both cases use will decrease; it is even the reason that in practice one generally gives the propellers, with constant pitch. a diameter not exceeding the pitch. Indeed, the pitch is a little larger than the advance A, so a wing which would have a length equal to half of the pitch, would be a little more than half of the advance per turn A, and if it is equal to 3 modules it will have a length $3M = 3\frac{A}{2\pi}$ so a little less than $\frac{A}{2}$, which is equivalent to about half the pitch. This explains the practical rule which consists of giving the propeller, at constant pitch, a diameter which does not exceed the pitch.

By averaging the retreat for the wing with a constant angle of attack, we find that between the limits taken for the normal wing, $r_0 = 0.5$, and $r_1 = 5$, the average of retreat will be:

$$H - A = 0.11A, \text{ about } 11\% \text{ of the advance.}$$

To draw the wing of the propeller at a constant angle of attack, there are two ways: the first one, the one we just saw and which consists in carrying on the radius ρ (fig. 7), from the origin 0, lengths respectively equal to 0.5 M, 1 M, 2 M, etc. ..., up to 5 M, when it comes to the normal wing, or more, if the calculation requires it; these values correspond to the values $tang\beta$ increasing from 0.5 to 5; we will thus obtain a series of points A, B, C, ... K. Then we will carry on the axis of X, from the origin, lengths respectively equal to $\frac{H}{2\pi}$ corresponding to the same increasing values of $tang\beta$; the values of $\frac{H}{2\pi}$ will be taken from Table C. It will get a series of points A '. B '. C '..., K', whose most recent next will be, not A ', but B' since the length OB' is minimum. We will join each of the points of the axis OX with the corresponding point of the radius, we will thus have a series of straight lines A'A, B'B, C'C, ... K'K, which will represent the inclination of the wing for each point of the radius. Taking on the line OX a point M, at the distance of the origin, such that OM is equal to the module, $OM = M = \frac{A}{2\pi} = \frac{V}{2\pi N}$, and joining this point to each of the points A, B, C, ... K, of the radius, the angles that will make these lines with the straight lines A'A, B'B, ... K'K represent the incidence under which each of the sections of the wing will meet the fluid threads; these incidemces are all the same and equal to the optimum incidence.

We will then determine the width of the wing at different heights A, B, C, ... K, as we will see below, and in fig.7. these templates have been (missing line here...) once the templates are cut, they are bent, following the radius ρ corresponding to the number of the template, and they are stored in their respective places on a board, to thus form the helical surface, on which the wing will be presented.

The second method, which is simpler, consists of carrying, as previously, the points A, B, C, ... K, on the radius, according to the increasing values of the module, from 0.5 to 5 (FIG. 8), then, take the length of the module on the X axis, to the point M, and lead the lines MA, MB, MC, ... MK. We will thus obtain the trace of a constant pitch, in which the pitch is precisely equal to the advance per revolution, since $M = \frac{A}{2}$ and at the same time are $\frac{H}{2\pi} = M$.

In this wing, the angle of attack is zero; if this wing turned, at its normal number of turns N, and advanced with the speed V in the direction of the axis OX, it would not exert any longitudinal thrust; so we could not use it as we have just sketched it.

Once this wing is executed according to the data above, it is at the moment of fixing it to its hub that it will be necessary to place it so as to obtain the constant angle of attack. For that, it will be enough to turn it around its principal radius, of an angle equal to the desired incidence, and to fix it definitively, with the hub of the tree, in this new staggered position. We understand easily this offset of the wing has the effect of rotating, from the angle α, all the elements, which were represented on our path (fig. 8) along the lines AM, BM, ... KM. If we redraw the new wing thus shifted, the new elements would be represented by straight lines making, with the old directions, angles α and intersecting with these old directions on the radius ρ. This new sketch would be precisely that which we indicated in the first method of drawing.

As generally, in aerial propellers, the wings are reported, this second method is preferable to the first, because it is simpler.

In our opinion, it is a mistake to attempt to give to the wings of aerial propellers the slight concavity usually given

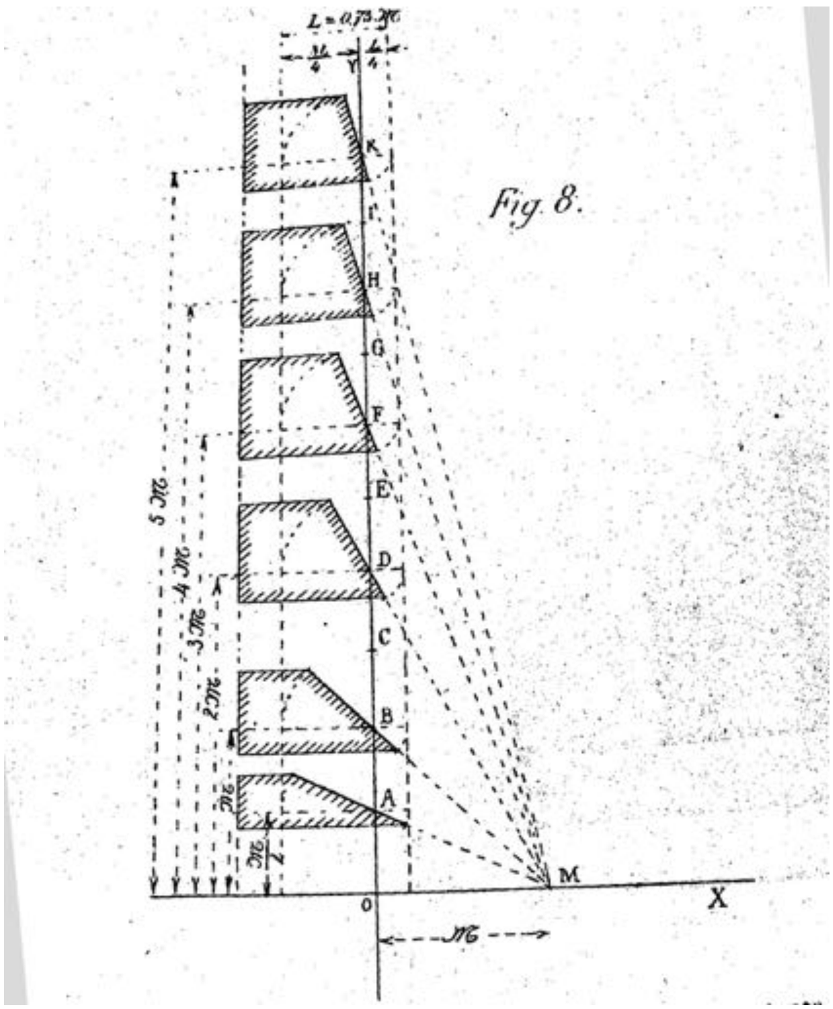


Fig. 8.

to the levelers, in order to increase their power capacity, the proportional decrease in weight worn; and moreover the strength of the device can only gain from it; we are left by increasing a little the power of the engine. While it is not the same for thrusters; their function consists in using, for propulsion, the power of the engine as best as possible: a diminution of the weight of the propeller has only an insignificant influence on the total weight, and on the contrary any increase of the harmful resistance even slight, leads, as we have seen, a very noticeable decrease in the use of propelling. The defenders of the concave wing system often invoke, in favor of their thesis, the argument that the air streams enter without impact on the surface of a concave wing, tangentially to the input element, and that these nets are deflected gradually to exit along the output generator, after having, by their reaction on the wing, produces the maximum thrust, similar to what happens in the curved-wing turbines. In our opinion, the two phenomena are not comparable; for in the turbines it is only a single bundle of isolated threads which strikes the curved wing, and it is in fact entirely deviated; while the wing of the propeller meets its full width parallel fluid threads, and if those entering tangentially through the generator of inputs can be progressively deflected by the concave surface of the wing, as in a turbine, the same is not true of all the other strings which strike the surface of the wing in all its width and especially towards the rear part, at angles of incidence increasing and greater than the optimum angle, which considerably reduces the efficiency of the thruster. That is why, until proven otherwise, we think it better to give the wings of propellers. flat helical surfaces, not concave, Once again, it is up to the aerodynamic laboratory to decide this question. If the laboratory tests showed that the ratio $\mu = \frac{f}{P}$, for hollow surfaces, is not greater than it is for flat surfaces, that is to say that the forces f and P grow in the same proportions, it is obvious then that it would have all the advantage to wings of hollow propellers.

In the above we have determined, for aerial propellers; 1○ the conditions of their maximum yield; 2○ the incidence to be given to the propeller elements; 3○ the dimensions that should be given to the length of the wings, and 4○ the way of plotting the pitch at the different parts of the wing; we have yet to determine the transverse dimensions of these wings, in other words, the width of the wing at the different radii.

For that, let us go back to the initial equations that serve to determine the motive power and useful power necessary to propel our airplane at the desired speed.
We have:

$$\zeta_M = PV\,(cos\beta + \mu sin\beta)\,tang\beta$$
$$\zeta_U = PV\,(sin\beta - \mu cos\beta)$$

Considering the elementary work per second we will have:

$$d\zeta_M = V\,(cos\beta + \mu sin\beta)\,tang\beta.dP$$
$$d\zeta_U = V\,(sin\beta - \mu cos\beta)\,dP$$

For a constant incidence α, the useful elementary thrust dP, will depend on the square of the speed with which the element in question encounters the gas molecules, the dimensions of the element and a empirical coefficient that we will call λ, we will have:

$$dP = \lambda W^2 ds$$

calling W the speed of the element with respect to the air and ds its surface. This presents a difficult question, it is the judicious choice of the coefficient λ. It is certain that it will be only through tests carried out in an aerodynamic laboratory that it becomes possible and even easy to determine exactly the exact value. This is the third time that, in the course of this study, we have come up against difficulties which only an aerodynamic laboratory is able to solve; therefore, the imminent need for the establishment of a laboratory of this kind can not be overemphasized, because it is only through this laboratory that we can determine the exact values of μ, α and λ, these three important parameters, without the exact knowledge of which, the establishment of, not only good aerial propellers, but also good airplanes, is absolutely impossible; until then it depends on more or less happy chances. In the absence of more precise data, we will adopt for λ a value which seems to respond quite well to reality, especially for the propeller wings, which are narrow and long surfaces attacking the air by their longest side; this value of λ would be, perhaps, a little strong for ordinary plans, especially if we use empirical formulas proposed by Colonel Duchemin, Hutton, Loïsel, M. Eiffel, etc., or a little weak, if we are to believe from other modern experimentalists, such as Langley, Maxime, Captain Ferber, and several other aviators. However, I think that we can adopt, without much error, $\lambda = 0.03$, expressed in kilograms, for one square meter of propeller wing, attacking the air under the optimum incidence, at a speed of 1 meter per second. As for the actual speed of the element, it will be $W = \dfrac{V}{cos\beta}$.

To determine the surface of the propeller element, we will call the width of the wing l and $d\rho$, the height of the slice considered along the radius ρ; we will have, for the expression of the elemental power:

$$d\zeta_U = \frac{\lambda V^3 \left(sin\beta - \mu cos\beta\right) l.d\rho}{cos^2\beta}$$

$$d\zeta_M = \frac{\lambda V^3 \left(cos\beta + \mu sin\beta\right) tang\beta l.d\rho}{cos^2\beta}$$

and for the total power, by calling a the number of wings:

$$\zeta_U = a.\lambda.V^3 \int_{r_0}^{r_1} \frac{sin\beta - \mu cos\beta}{cos^2\beta} l.d.\rho$$

$$\zeta_M = a.\lambda.V^3 \int_{r_0}^{r_1} \frac{cos\beta + \mu sin\beta}{cos\beta} tang\beta.l.d\rho$$

It is by means of these equations that we will be able to determine l.

Theoretically it can be done indifferently either from the expression of the useful power or from that of the motive power: however, in practice, it is preferable to base oneself on the motive power; it is the only one we know, since it can be measured directly on the engine shaft, while for the useful power it must be deduced based on the performance of the thruster that we do not always know.

We will therefore adopt this first method of calculation and we will seek to deduce the width l, from the expression of the motive power.

In the equation of the motive power, replace $tang\beta$ by its value $\frac{2\pi N\rho}{V}$, and divide the numerator and the denominator by $cos\beta$, we shall have:

$$\zeta_m = 2\pi N\alpha\lambda V^2 \int_{r_0}^{r_1} \frac{1+\mu tang\beta}{cos\beta}.l.\rho.d\rho.,$$

In this expression, ζ_M denotes the engine power in kilograms on the propeller shaft, a the number of wings, N the number of revolutions of the propeller per second, V the speed of advancment of the airplane, in meters per second, and l the width of the wing in meters at the different radii ρ, corresponding to the successive values of $tang\beta$. The width l is the corrected intersection of the surface of the wing by a cylinder of radius ρ, and whose axis would be the axis of rotation of the thruster. To derive from this equation the different values of l, corresponding to the successive and increasing values of $tang\beta$, we must link the variables l and ρ by a relation of dependence chosen by the consideration of the form to be given to the wing; this form can be modified at will by modifying these dependency conditions. We can, for example, adopt, as a condition, that the transversal thrusts experienced by the different cylindrical sections of the wing, represented by the helical bands of length l and height $d\rho$, be distributed along the radius according to a function of this radius such that $C\varphi(\rho)$, or C, is a parameter to determine. The elementary thrust being expressed by $C\varphi\,(\rho)\,\rho d\rho$, the power of the engine torque will be:

$$\zeta_M = 2\pi N.\alpha \int_{r_0}^{r_1} C.\varphi\left(\rho\right).\rho.d\rho.,$$

on the other hand we have the expression of this same motive power:

$$\zeta_M = 2\pi N\alpha\lambda V^2 \int_{r_0}^{r_1} \frac{1+\mu tang\beta}{cos\beta} l.\rho.d\rho,$$

let's match these two expressions:

$$C_{\varphi}\left(\rho\right) = \lambda.V^2 l\frac{1+\mu tang\beta}{cos\beta},$$

from which:

$$l = \frac{C.\varphi\left(\rho\right) cos\beta}{\lambda.V^2\left(1+\mu tang\beta\right)} \ .$$

To determine the parameter C, let us deduce it from the previous equation:

$$C = \frac{\zeta_M}{2\pi N\alpha \int_{r_0}^{r_1} \varphi\left(\rho\right).\rho.d\rho},$$

and replacing C by this value in the expression of l, one has:

$$\mathrm{l} = \frac{\zeta_M cos\beta\varphi\left(\rho\right)}{2\pi.N.\alpha.\lambda.V^2\left(1+\mu tang\beta\right)\int_{r_0}^{r_0} \varphi\left(\rho\right).\rho.d\rho},$$

replace ρ by its value ρ= $\frac{Vtang\beta}{2\pi N}$, in addition, let $\frac{\zeta_M}{75} = F$, which will express the power of the engine in horsepower on the axis of the propeller, replace λ by its value $\lambda = 0.03$ and π by 3.14, let us perform the calculations, we find:

$$l = \frac{15494F.N}{\alpha V^4 \int_{tang\beta_0}^{tang\beta_1} \varphi\left(tang\beta\right) tang\beta.dtang\beta}.\frac{cos\beta}{1+\mu tang\beta}\varphi\left(tang\beta\right)$$

There is an infinity of functions $\varphi(tang\beta)$ that can be chosen to determine the shape of the wing, There is one particular very interesting and that can have a direct application for the wings of aerial propellers; it is the one where the width l remains constant all along the wing. We will call this special width a *specific width* and we will designate it by the letter L.

To do this we must find a function which makes the value of L constant.

To satisfy this condition will require that the variable $\frac{\varphi(tang\beta)cos\beta}{1+\mu tang\beta}$ is equal to unity, or:

$$\varphi\left(tang\beta\right) = \frac{1+\mu tang\beta}{cos\beta} = \left(1+\mu tang\beta\right)\sqrt{1+tang^2\beta},$$

which will give us:

$$L = \frac{15494FN}{\alpha V^4 \int_{tang\beta_0}^{tang\beta_1} \left(1+\mu tang\beta\right)\sqrt{1+tang^2\beta}.tang\beta d\left(tang\beta\right)},$$

To make integration easier, Let us say:

$$\gamma = \sqrt{tang^2\beta + 1} + tang\beta,$$

therefore:

$$\gamma_0 = \sqrt{tang^2\beta_0 + 1} + tang\beta_0 \text{ and } \gamma_1 = \sqrt{tang^2\beta_1 + 1} + tang\beta_1,$$

which will give us, after integration between the given limits γ_0 and γ_1:

$$L = \frac{247820.F.N}{\alpha V^4\left[\frac{\mu}{4}\left(\gamma_1^4 - \gamma_0^4 - \frac{1}{\gamma_1^4 - \gamma_0^4}\right) + \frac{2}{3}\left(\gamma_1^3 - \gamma_0^3 + \frac{1}{\gamma_1^3 - \gamma_0^3}\right) + 2\left(\gamma_1 - \gamma_0 + \frac{1}{\gamma_1 - \gamma_0}\right) - 2\mu\zeta\frac{\gamma_1}{\gamma_0}\right]}$$

For the normal wing for which $r_0 = 0.5$ and $r_1 = 5$, taking $\mu = 0.05$, we will have, in doing the calculations:

$$L = 298.5\frac{FN}{\alpha V^4}\ .$$

We know that in the case of an elongated rectangular flat surface, attacking the air at a small angle, the resistance is much greater when the air is attacked by the wide width than if it was by the small. This is explained to a certain extent by the consideration that, in the attack by the small side of the rectangle, the air can more easily flow through the long sides of the pallet, while when it is the big side that attacks the air, the lateral flow is much reduced, since it is proportional to the length of the slice, and that slice, in this second case, is smaller. so it will be advantageous that the ratio of the width of the wing to its length is low: take for example the constant ratio of $\frac{1}{6}$ between the width of the wing and its length, it is a report which seems to give good results in the practice.

So let us ask $\dfrac{L}{r_1 - r_0} = \dfrac{1}{6}$; but we know that:

$$r_1 - r_0 = \frac{V}{2\pi N}\,(tang\beta_0 - tang\beta_0),$$

and for:

$$tang\beta_0 = 0.5 \text{ and } tang\beta_1 = 5,$$

we will have:

$$r_1 - r_0 = \frac{V}{N}0.717\ .$$

Division L by the value found for $r_1 - r_0$ we will have:

$$\frac{L}{r_1 - r_0} = 416.8\frac{FN^2}{\alpha V^5}\ .$$

As, on the other hand, we have assumed that in normal propellers the ratio $\dfrac{L}{r_1 - r_0}$, should be $\frac{1}{6}$, replacing this ratio by its value we will have:

$$1 = 2500\frac{FN^2}{\alpha V^5}$$

from where:

$$a = 2500\frac{FN^2}{V^5}\ .$$

We will name this expression the *equation of compatibility*, because it will be used to determine the compatibility of the four quantities involved, the power in HP, the number of revolutions N from the thruster per second, the velocity V of the airplane in meters, and α, the number of blades of the propeller. When we have an aerial propeller to be determined, we will have to start from a base, this base will be first of all the power necessary to remove the airplane and the speed of its advance; these data are independent of the propeller, starting from these data it will be necessary to calculate the propeller so that its number of revolutions to the second and its equation of compatibility is satisfied. The number of wings thus determined will obviously relate to normal wings. If the operating conditions of the propeller were such that the compatibility equation could not be satisfied, that is, the number of wings found by the computation was too great, or did not give an integer, it will be necessary either to increase the length of the wings, as we shall see later, or to increase their width, in proportion to the number found by the calculation, to the adopted figure.

It must be realized that this compatibility equation is a perfectly legitimate convention and that we have all the right to do; it simply means that the number of normal wings, computed by the equation of compatibility, running at a number of revolutions N per second, and advancing in the air with a velocity of V meters to the second, absorb a motor power of horsepower; moreover, that the specific width of these wings is equal to the sixth of the length of the wing.

Let us take again the expression of the specific width which we have just found:

$$L = \frac{298.5FN}{\alpha V^4},$$

divisions this width by the module $M = \dfrac{V}{2\pi N}$, we will have:

$$\frac{L}{M} = 1875\frac{FN^2}{\alpha V^5}\ .$$

Here we find the expression we have just seen in the compatibility equation.

Let us replace, in $\frac{L}{M}$, this expression by its numerique value:

$$\frac{L}{M} = \frac{1875}{2500} = 0.75$$

then

$$L = 0.75M$$

which means that for a normal wing, to which a constant specific width has been given, this specific width will be the 3/4 of the module.

This value of the specific width can also be deduced from the ratio we have assumed between the specific drop and the length of the wing, which is 1/6 as the length of the wing is 4.5 modules, the sixth which represents the specific width will be

$$L = \frac{4.5}{6}M = 0.75M$$

The wing shape with constant specific width L, will be one of the best to use for aerial propellers; it will be a rectangle whose height is 4.5 times the module, and its width, equal to 3/4 of the module the wing will start at a distance of half a module of the axis and its radius will be 5 modules.

In this way the normal wing with constant specific width is completely determined in all its elements, all of which are expressed in abstract digits, because they all have the module for common scale.

For the general expression of the specific width express in module, it will be necessary to divide the general expression of L by M and replace the term $\frac{FN^2}{\alpha V^5}$ by its numeric value $\frac{1}{2500}$ at that we will then have:

$$\frac{L}{M} = \frac{622.7}{\frac{\mu}{4}\left(\gamma_1^4 - \gamma_0^4 - \frac{1}{\gamma_1^4 - \gamma_0^4}\right) + \frac{2}{3}\left(\gamma_1^3 = \gamma_0^3 + \frac{1}{\gamma_1^3 - \gamma_0^3}\right) \over +2\left(\gamma_1 - \gamma_0 + \frac{1}{\gamma_1 - \gamma_0}\right) - 2\mu\zeta\frac{\gamma_1}{\gamma_0}}$$

If, according to this formula, the values of $\frac{L}{M}$ are calculated by $r_0 = 0.5$ and r_1 successively equal to 5, 6, 7, 8, modules, we find the figures:

for r_1=	5M,	6M,	7M,	8M
L=	0.75M,	0.427M,	0.275M,	0.175M

We see that the specific width, for r_1= 5, is 0.75 modules, that is to say 1/6 of the length of the wing; for $r_1 = 6$, this specific width is only $\frac{1}{13}$, for $r_1 = 7$ it is $\frac{1}{25}$ and for $r_1 = 8$ only $\frac{1}{44}$ of the length of the wing. So if we wanted to, for all these wings have the ratio of 1/6, assumed for normal wing, we will have to increase the length of the second wing in the ratio of 2.1, the third in the ratio of 4.1, and the last in the ratio of 7.3. Therefore a propeller having a radius $r_1 = 8$, and whose wing width is 1/6 of the length, as in normal wings, would have an active propulsive surface 7.3 times too large. Also, when the equation of compatibility shows that the number of wings required a, is greater than the number we wish to adopt, and which will have to be multiplied by the ratio $\frac{a}{a'}$, the wing widths used, we can always find an outer radius r_1, greater than 5, such as the specific width of the wing, divided by the length of the wing $\frac{L}{r_1 - r_0}$, and multiplied by the ratio $\frac{a}{a'}$, exactly equal to $\frac{1}{6}$, the ratio allowed for normal wings.

So if the ratio $\frac{\alpha}{\alpha'}$ was for example 7.3 then we would have to give to r_1 a value of 8 M, and a wing width equal to $\frac{1}{6}$ of its length; the number $a^{'}$ of wings, thus modified, would be equivalent to normal wings, that is to say, would absorb the driving power F, turning at N turns, and advancing at the speed V. Let us take the general expression of the width of the wing:

$$l = \frac{15494FN}{aV^4 \int_{tang\beta_0}^{tang\beta_1} \varphi\left(tang\beta\right) tang\beta.d\left(tang\beta\right)} . \frac{cos\beta}{1+\mu tang\beta} \varphi\left(tang\beta\right).$$

Divide by M $= \frac{V}{2\pi N}$, the two members of the equation, we will have:

$$\frac{l}{M} = \frac{97290FN^2}{aV^5 \int_{tang\beta_0}^{tang\beta_1} \varphi\left(tang\beta\right) tang\beta.d\left(tang\beta\right)} . \frac{cos\beta}{1+\mu tang\beta} \varphi\left(tang\beta\right)$$

We have just seen that in the compatibility equation $\frac{FN^2}{aV^5} = \frac{1}{2500}$, we can therefore replace this expression by its numerical value and it comes:

$$\frac{l}{M} = \frac{38.9}{\int_{tang\beta_0}^{tang\beta_1} \varphi\left(tang\beta\right) tang\beta.d\left(tang\beta\right)} . \frac{cos\beta}{1+\mu tang\beta} \varphi\left(tang\beta\right),$$

or, to abbreviate the notation, designate by $\varsigma = tang\beta$. by $\varsigma_0 = tang\beta_0$ by $\varsigma_1 = tang\beta_1$ and $\varsigma^{'} = cos\beta$, there will be:

$$\frac{l}{M} = \frac{38.9}{\int_{\varsigma_0}^{\varsigma_1} \varphi\left(\varsigma\right) .\varsigma.d\varsigma} . \frac{\varsigma^{'}}{1+\mu\varsigma} \varphi\left(\varsigma\right),$$

an expression independent of power F, the number of turns N and of the velocity V, and depending only on the various values of ς. It is understood that this expression is true only if the conditions determined by the compatibility equation are assumed.

It is by means of this equation that one determines the number of wings necessary for the given conditions. However it could happen that the number of wings calculated differs from the one that could be used, in practice, for the given case; then, if the difference between the two numbers were small, we would be content to multiply the widths of wings, than we will have calculated by means of one of the formulas above, by the ratio of the number of wings that will determine the equation of compatibility and the number of real wings, We will call this ratio $q = \frac{a}{a'}$ the *coefficient of reduction*; by calling the number of wings calculates and the real number, In the case where the ratio q becomes too large, close to 2 for example, or above all higher, it would no longer be possible to apply this method, since the lengths wings thus obtained would become more than $\frac{1}{3}$ their length, which for the wings of an aerial propeller would be exaggerated; it would be appropriate then to give up the wing we have adopted for the normal wings and that is 5M, and increase this length up to 6M, 7M, 8M, and perhaps beyond that. It would be necessary in this case to calculate the wing widths directly by the general formulas, giving to ς the adopted value. Thus, for the case of the computation of the specific width L we will take:

$$\frac{L}{M} = \frac{1556310.F.N^2}{\alpha V^5 \left[\frac{\mu}{4}\left(\gamma_1^4 - \gamma_0^4 - \frac{1}{\gamma_1^4 - \gamma_0^4}\right) + \frac{2}{3}\left(\gamma_1^3 = \gamma_0^3 + \frac{1}{\gamma_1^3 - \gamma_0^3}\right) + 2\left(\gamma_1 - \gamma_0 + \frac{1}{\gamma_1 - \gamma_0}\right) - 2\mu\zeta\frac{\gamma_1}{\gamma_0}\right]}$$

by doing:

$$\gamma_0 = \sqrt{\varsigma_0^2 + 1} + \varsigma_0, \text{ and } \gamma_1 = \sqrt{\varsigma_1^2 + 1} + \varsigma_1.$$

We will increase the value of ς_1, until the width obtains L, is about $\frac{1}{6}$ or $\frac{1}{5}$ of the length of the wing which is $\varsigma_1 - \varsigma_0$ always assuming for ς_0 the value $\varsigma_0 = 0.5$.

If one wanted to give the wing, not the form of equal specific width, but a different form, one would use the general formula:

$$\frac{l}{M} = \frac{97290F.N^2}{a.V^5 \int_{\varsigma_0}^{\varsigma_1} \varphi(\varsigma).\varsigma.d(\varsigma)} . \frac{\varsigma'}{1 + \mu\varsigma} . \varphi(\varsigma),$$

by choosing, for $\varphi(\varsigma)$, a function which gives the wing the desired shape.

One could also, as we have shown above, adopt for; ς_1 a value such that the corresponding specific width, multiplied by the reduction coefficient q, is about $\frac{1}{6}$ or $\frac{1}{5}$ of the length of the wing, So, we have seen that for the values of ς_1 equal to 6M, 7M, and 8M the corresponding specific widths were $\frac{1}{13}$, $\frac{1}{25}$ and $\frac{1}{44}$ of the length of the wing, respectively; Therefore. if we multiply these widths by a coefficient q, which would be, for the first wing, of 2.1, for the second, of 4.1. and for the last, 7.3, we would still obtain a wing whose width would not exceed $\frac{1}{6}$ of its length.

We have shown that, under the conditions of satisfying the equation of equivalence, wing widths could be calculated by the general formula:

$$\frac{l}{M} = \frac{38.9}{\displaystyle\int_{\varsigma_0}^{\varsigma_1} \varphi(\varsigma).\varsigma.d(\varsigma)}.\frac{\varsigma^{'}}{1+\mu\varsigma}.\varphi(\varsigma),$$

and that properly choosing the function $\varphi(\varsigma)$ could achieve the wing shape that was desired.

We will review a number of these functions which will give us wing shapes applicable to aircraft thrusters, either directly or in combination with other functions.

One of the simplest is $\varphi(\varsigma) = \varsigma + p$, where p is an arbitrary parameter that can be varied at will.

By replacing $\varphi(\varsigma)$ with the function adopted, we obtain after integration:

$$\frac{l}{M} = \frac{233}{2(\varsigma_1^3 - \varsigma_0^3) + 3p(\varsigma_1^2 - \varsigma_0^2)}.\frac{\varsigma^{'}}{1+\mu\varsigma}.(\varsigma + p),$$

and for the normal wing, between the limits $\varsigma_0 = 0.5$ and $\varsigma_0 = 5$:

$$\frac{l}{M} = \frac{233}{249.7 + 74.3p} \cdot \frac{\varsigma'}{1 + \mu\varsigma} (\varsigma + p) \ .$$

For example, to show the most convenient way to group calculations, we have, in table D, presented the calculation widths of a normal wing $\frac{l}{M}$, by the formula above, adopting for p the value $p = 1$.

Tableau D

Valeurs successives de s.....	0,5	1	2	3	4	5
Valeurs correspond. de $(s+p)$.	1,5	2	3	4	5	6
Logarithmes de ces valeurs..	0,1761	0,3010	0,4771	0,6021	0,6990	0,7782
Log. de $\frac{s'}{1+\mu s}$ (tirés du tableau de l'appendice)......	$\bar{1}$,9408	$\bar{1}$,8283	$\bar{1}$,6091	$\bar{1}$,4392	$\bar{1}$,3055	$\bar{1}$,1953
Log. du facteur $\frac{233}{249,7 + 74,3}$	$\bar{1}$,8569	$\bar{1}$,8569	$\bar{1}$,8569	$\bar{1}$,8569	$\bar{1}$,8569	$\bar{1}$,8569
Log. $\frac{l}{M}$ (sommes des trois rangées)...	$\bar{1}$,9738	$\bar{1}$,9862	$\bar{1}$,9431	$\bar{1}$,8982	$\bar{1}$,8614	$\bar{1}$,8304
Valeur correspondante de $\frac{l}{M}$	0,942	0,969	0,877	0,791	0,727	0,677

In the first horizontal line, we have arranged the successive values of ς, from $\varsigma = 0.5$ to $\varsigma = 6$; below, we have given the corresponding values of $\varsigma + p$ assuming $p = 1$. In the third line come the logarithms of the numbers of the preceding row. Below are the logarithms of the variable factor $\frac{\varsigma'}{1 + \mu\varsigma}$, whose successive values have been calculated once and for all from $\varsigma = 0.5$, up to $\varsigma = 8$. These values, which appear in the table E of the appendix have been calculated for values of ς varying by one unit up to $\varsigma = 5$ and by half a unit from $\varsigma = 5$ to $\varsigma = 8$. This has been made with a view to the possibility of adopting for ς, greater than 5M, an intermediate value between two integer values of ς: and in this case the logarithm corresponding to one of these intermediate factors proved to be necessary. In the fifth horizontal row we say again in each column. the logarithm of the constant factor $\frac{233}{249.7 + 74.3p}$ which for p = 1, gives the logarithm of 1.8569.

By making in each column the sum of the logarithms of the three factors $\varsigma + p, \frac{\varsigma'}{1 + \mu\varsigma}$, and the constant factor, we obtain the logarithms of widths $\frac{l}{M}$. which are the numbers in the sixth row. Below these logarithms are arranged the actual values of the widths sought.

In the figure 9 we have represented graphically the curves which give the widths of wings calculated by the formula above and in which we successively varied p. since $p = 0$, $p = 1$, $p = 2$, up to p infinity, which corresponds to $\varphi(z)$ = constant.

Looking at this figure, we see that the wing, determined by the curve that corresponds to $p = 0$, is very narrow at the bottom and widens towards the end. By increasing the value of p, the wing progressively widens at its origin and retracts at its end; for $p = 1$, the wing shape obtained is directly applicable to the air thrusters. With the increase of p, the wing widens considerably at the bottom and becomes very narrow towards its end. This form of wing can no longer be used in practice, in its current state, but could be combined with another form which would, on the contrary, be narrow at the bottom and broad towards the end. In this case half of the widths would be calculated by means of one formula, and the other half by means of another suitably chosen one, and the sums would be made.

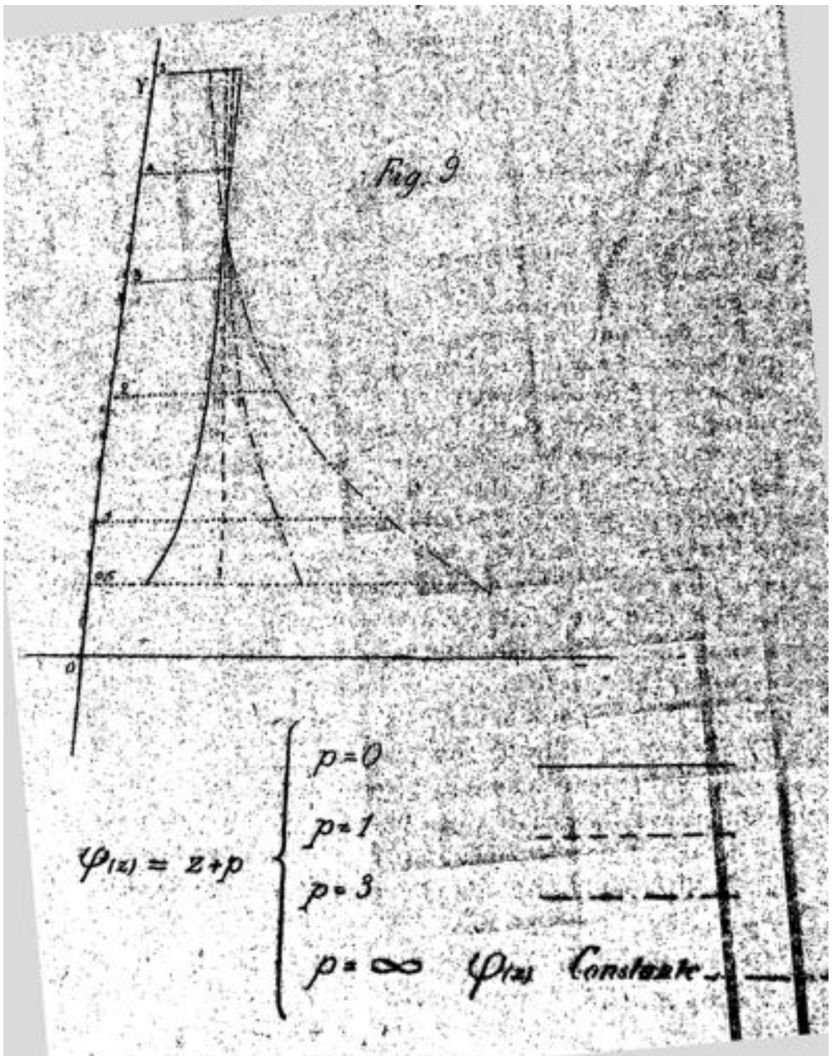
We wanted to indicate how the shape of the wing changed with the variation of the arbitrary parameter, to show the ease with which we can obtain a desired wing shape by appropriately choosing the function $\varphi(z)$ and the arbitrary parameters.

In figure 10 we have grouped the curves obtained by another function:

$$\varphi(z) = z^2 + p.$$

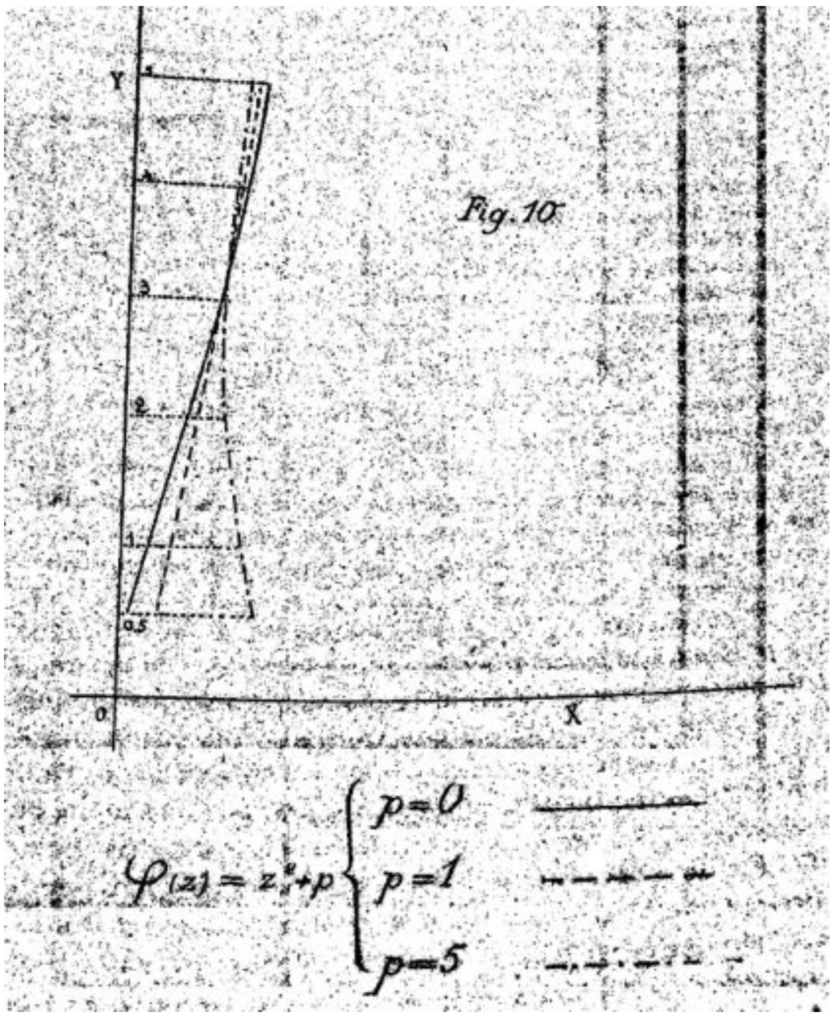
In that case; the deviant equation:

$$\frac{l}{M} = \frac{155.6}{(z_1^4 - z_0^4) + 2p\,(z_1^2 - z_0^2)} \cdot \frac{z^{'}}{1 + \mu z}\,\big(z^2 + p\big),$$



and for the normal wing between the limits z0 = 0.5 and z1 = 5.

$$\frac{l}{M} = \frac{155.6}{624.9 + 49.5.p} \cdot \frac{z^{'}}{1 + \mu z} \left(z^2 + p\right)$$



The three curves in figure 10 have been calculated by successively giving p the values of $p = 0$, $p = 1$ and $p = 5$.

The first of these curves is very narrow at the bottom and widens upwards; the second is less accentuated than the first and the third narrows the wing slightly towards its center. This last form could be combined with a form which, on the contrary, would widen a little towards its middle: one could arrive there, for example, by the function $\varphi(z) = pz^2 - z^3 + q$ in which p would be greater or equal to $z_1$and q arbitrary:

One would then have:

$$\frac{l}{M} = \frac{778}{5p\left(z_1^4 - z_0^4\right) - 4\left(z_1^3 - z_0^3\right) + 10.q\left(z_1^2 - z_1^2\right)} \cdot \frac{z^{'}}{1 + \mu z}\left(pz^2 - z^3 + q\right)$$

One could vary to infinity the choice of various functions; we will, however, confine ourselves to those we have passed in review, because they are amply sufficient for the calculation of the wing widths of an aerial propeller, especially since the shape of an aerial propeller will always have to be close to a common type, generally long and narrow,

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Of all the forms of wings that can be adopted for an aerial propeller, it is still that of constant specific width which is the most convenient to calculate and which will most often find its direct application. It also has the advantage of facilitating the comparison of the different propellers with each other.

The wings we have just reviewed are all the same length, of 5M, they rotate at the same number of revolutions N, advance with the same speed V, the same motive power F: they are therefore all *equiactive*, and their propulsive surfaces have the same mechanical *efficiency*, notwithstanding the very notable differences in their geometrical dimensions. This shows how much the *active* surface of the wing is different from the geometrical surface. Also, when we say that it is appropriate to give the propeller such a propellant surface, it means nothing at all, as long as we have not indicated the *distribution* of this surface. This is easy to understand. For a square centimeter of propellant surface will have a very different mechanical action depending on whether it is placed in the vicinity of the axis of rotation or towards the end of the wing, the thrusts produced being proportional to the squares of the speeds and therefore the radii.

In addition, the inclination of this propulsive element will be different according to its position on the wing and, therefore, it will absorb a different motive power depending on whether it is more or less inclined.

It follows that it is impossible to determine the *active* surface of a helix by its geometric surface, or its average pitch fraction, as is very often done by ignorance or routine.

Whenever, for a wing, we want to determine the active surface, it is essential to deduce from the mechanical elements of its operation. Also, we constantly see absolutely equiactive wings, differ considerably from the point of view of their geometrical surface, and reciprocally from the wings of the same geometrical surfaces have absolutely different mechanical effects.

All the wings that we have just reviewed are rigorously equiactive and yet, as can be seen in figures 9 and 10, they differ very substantially in terms of their geometric surface. They are, moreover, all equiactive with the wing with constant specific width in which the width L is equal to $\frac{3}{4}$ of the module. So here is a way to compare the different propellers with each other; it is sufficient to replace them by their equiactive form, with a special width constant, and the ratio of these specific widths will rigorously give the ratio of the active surfaces of these propellers.

This is true, not only for propellers of the same diameter, but also for those of different diameters, provided that one has to deal with normal wings.

Because, for normal wings, we know that the specific width is equal to 0.75M, we will immediately have the equi- active form of the given wing, if, however, at construction, the wing widths have not been multiplied by a reduction coefficient. It will be easy to be convinced of this by doing the *mechanical* (and not geometrical) averaging, the widths of the given wing, corresponding to the radii of the wing of a multiple of module.

For this we will sum the products of these widths by the squares of the corresponding radii and we will divide it by the sum of the edges of the radii;

we will thus have the average of mechanical width which corresponds to the specific width: this average will be:

$$L = \frac{\sum l.\rho^2}{\sum \rho^2} \ .$$

It is understood that in this equation all these quantities are expressed as a function of the module, therefore in abstract numbers.

When the value of the specific width of the wing has been increased, it will be easy to compare it with the normal specific width if the adopted widths have not been multiplied by a coefficient of reduction, and we find this coefficient if it was used.

This mode of comparison of the wings of the propeller, which consists in transforming the wing into an equiactive wing with a constant specific width, can even be applied to the wings with variable angle of incidence, as are the wings with constant pitch. Only in this case will it be necessary to take into account for each band, which represents the width of the wing at the given radius, its own incidence: and since the thrusts are substantially proportional to the incidences, it will be necessary, in the mechanical average, to take into account these incidences by posing:

$$L = \frac{\sum l \dfrac{\alpha^{'}}{\alpha}.\rho^2}{\sum \rho^2},$$

$\alpha^{'}$ being the actual incidence of the band whose width is l. and α constant of optimum incidence. This mechanical average is sufficiently accurate. This will result in a wing with constant incidence and constant specific width, completely equiactive with the wing of variable incidence and of any shape. The constant specific width can therefore be used for all the propeller wings.

It is understood that for all wings, whatever they are, we assume once and for all the radius of the hub $z_0 = 0.5$.

We have seen that all elements of the wing, without exception, express themselves according to this common measure which is the modulus: moreover, for the normal wings, all these elements are constant and do not vary from one propeller to another; thus the spokes of the hub and the wing are always expressed by:

$r_0 = 0.5M$ and $r_{1=5}M$,

the lengths taken on the intermediate radii are always the same:

$$0.5M,\ 1M,\ 2M,\ 3M,\ 4M \text{ and } 5M,$$

the wing widths, at these radii, are also the same, whether for the specific width, which is $0.75M$, or for the widths calculated by one of the formulas above; they will always be the same for the same radii. As for the variable pitch of the wing, it is also always the same expressed according to the module, for the same point of the radius. In this way, it can be said that there is only one normal wing whose dimensions depend on the size of the module. If, then, we calculate once for all the elements of a normal wing, this calculation will serve for all the normal wings which may exist; the same will be the case with the normal wing, which will serve indifferently for all normal wings; only the scale will change. This scale is the modulus, it depends on the operating conditions at the given wing since it depends on the speed and the number of revolutions. With regard to the number of wings to be used for each case, this number is determined, as we have seen, by the equivalence equation. So we see that we can calculate and plot any propeller using only the equation of compatibility and the table computed, once and for all, which will determine the elements of the wing according to its module.

In most cases, the normal wing should be able to be applied, thanks to a judicious choice of the elements that will be adopted in the compatibility equation. In the case, however, where this equation could not be satisfied, it would be enough to increase the radius of the wing.

So we come to this conclusion that the thruster is not strictly speaking, the propeller, but that it is the wing which is the organ of propulsion and that this organ is always the same, albeit on a different scale: it is not that the number of these organs must be greater or less, according to the motive power to be absorbed, and that is what determines the equation of compatibility.

This, then, is the final result to which we have been led on the basis of a perfectly correct and legitimate postulate: this postulate has been the determination of the resistance experienced by a plane element moving along a helical path and meeting the fluid threads under a plane, certain incidence. In the whole series of our reasonings, we have made no hypothesis, and we have been led by a series of logical deductions and rigorous computations to the conclusions which we have just formulated. We can thus affirm without fear that the present theory of helical thrusters is strictly accurate, at least qualitatively, because from the quantitative point of view, it obviously depends on the coefficients adopted in the course of this study. If their numerical values were different from those which we assumed, nothing would be changed in the theory, and it would suffice to modify the results of the calculations in proportion to these new coefficients. So, to obtain absolutely correct results, with the propeller conception that we possess thanks to this theory, we have only to determine in a precise way and, once and for all, the numerical values coefficients that we used. This determination can only be made in a rigorous manner in an aerodynamic testing laboratory.

Elsewhere we have indicated how to organize a laboratory of this kind.

It should consist of a large diameter tunnel in which an artificial air current would be circulated at a speed equal to that which could be achieved by an aircraft in calm air. It is in this current of air that one would install, with stationary station, the devices to be experimented, such as levellers and thrusters. These devices should be of natural size. Propulsive propellers should be tested as follows. A dynamometric scale with an electric motor should be installed in order to record directly the resistance torque, the number of revolutions and the longitudinal thrust. It is easy to imagine a scale of this kind, as rigorous as one would like; one could even take as a model the one that Colonel Renard had built for liftimg propeller testing at Chalais-Meudon. On the rotating shaft, the propeller to be tested would be installed, the air flow in the tunnel would be regulated at a determined speed V, and the wings of the propeller would be constructed so as to have a constant pitch exactly equal to the lead per turn which is $\frac{V}{N}$, where N is the number of turns which should rotate the propeller shaft. When the propeller would operate under these conditions, the angle of attack would obviously be zero, since the pitch would be equal to the advance; in this case, the longitudinal thrust would be zero, and all the motive power would be expended to overcome the friction of the air on the wings. The value of the resistive torque and the number of revolutions of the helix would give the measure of the power absorbed. Simultaneously varying the speed of the tunnel airflow and the rotational speed of the propeller, so as to maintain the constancy of the ratio $A = \frac{V}{N}$, we would have, under these different conditions, the values of the power absorbed by the friction of the wings in the air. The curve of these powers would be drawn up as a function of the increasing speed. This first series of tests would be intended to determine one of the parts of the ratio μ which, as we have shown, consists of a term that is $tang\alpha$. and another which is due to the friction of the wing in the air. In this first experiment, α being zero, it would be the second term alone that we would measure. After this first series of tests, the wings of the propeller would be shifted on the hub, rotating them by 1 degree around their main radius and in the direction of pitch increase. This offset would give the wings an angle of attack $\alpha = 1^\circ$, that the advance per revolution remains the same as before, that is to say that the number of revolutions of the propeller is always proportional to the velocity of the air current. In this new series of tests, we will obtain a longitudinal thrust that we measure for each speed, This thrust, multiplied by the speed of the current, will represent the useful power, and the ratio of the useful power to the motive power, recorded directly, will give the utilization coefficient K. In addition, the resistive torque will give the measure of the harmful resistance f, while the longitudinal thrust will measure the useful thrust P. The ratio of these two forces will determine the numerical value of μ.

After this second series of experiments, the wings of the helix will be shifted further by one degree, so as to increase the angle of attack and bring it to 2 °, still under the same conditions of advance per turn. We will redo a new series to determine the yield curve. After which we will shift the wings to 3 ° and we will start the same tests, Comparing the yield curves in these three series of tests, with 1 °, 2 ° and 3 ° from the angle of attack, we will see if the yields increase or diminish. If they increase, the angle of attack will be further increased until we reach the maximum yield. At this moment, by bringing the limits of the variations of the angle of attack closer, it will be possible to determine rigorously and once and for all the really optimum angle of attack. At the same time, the average maximum efficiency of a propeller wing will also be determined. We will then proceed to the verification of the coefficient λ. For this, we will experiment with normal wings with constant specific width equal to $\frac{3}{4}$ of the module. The propeller will be rotated by the number of revolutions N in an air stream of velocity V, and the motive power expended will be recorded. If this motive power is weaker or stronger than that indicated by the calculation, for the given case, it is because the real λ coefficient is stronger or weaker than that which we have assumed; and it will be necessary to correct the assumed value $\lambda = 0.03$, by multiplying it by the direct ratio of the motor torque, calculated and real, since we know that if:

$$P = \lambda.s.W^2.\alpha$$

so if:

$$P' = \lambda'.s.W^2.\alpha,$$

we will have the direct report:

$$\frac{P}{P'} = \frac{\lambda}{\lambda'},$$

calling λ' the corrected coefficient and P' the measured thrust;

so:

$$\lambda' = \frac{P}{P'}\lambda\ .$$

This series of tests will allow us to determine rigorously and once and for all this coefficient λ on which the width of the wings depends, As a verification, we experiment with a second propeller, in all respects similar to the first one, but in which the specific width will have been modified in the ratio of $\frac{\lambda'}{\lambda}$, so we will find, in the new test, that the absorbed engine power is equal to that calculated.

After this series of experiments, which are in short very easy to undertake, all the quantitative elements of our theory will be rigorously known, and it will then become possible to calculate, without the slightest error, propulsive propellers, of maximum yield, for all given cases.

It would be quite unrealistic to attempt this kind of testing outside the laboratory, or as it is generally believed to be able to determine the coefficients in question using results provided by the continuous practice of aviation appliances and llitle by llittle improving the propellers. We do not believe that a method of this kind can give satisfactory results, since the measurements of the elements of the problem are not possible in practical exercises, that is why we can not emphasize enough the need for a laboratory.

As for the tests of propellers at the fixed point, as they are generally done, this method is only possible for lift propellers, for others, it is absolutely false and the results obtained by these tests have nothing in common with those that would be found if the propeller worked in the actual conditions of speed and number of revolutions. In the fixed point and running tests, the driving power, the number of revolutions and the longitudinal thrust are absolutely different in each case.

In the case of lift propellers, we will not deal with them in this study because they fall less in the category of thrusters than in that of reaction fans. It does not seem possible for us to treat this question by the same method that we have followed for the helicoidal thrusters, because for lifting propeller the phenomena are quite different; we can not follow, as in the case of thrusters, the trajectory of fluid threads, because we do not know their direction.

In front of the propeller, which turns at the fixed point, there is an aspiration of air, and the fluid threads arrive on all sides to fill the depression produced, so it is impossible to determine under what angles of attack they encountered the wings of the lifting propeller.

At first glance, it would seem rational to use the power of the motor as much as possible and to obtain the maximum of lift, to seek to have minimum angles of attack, close to the optimum angle. Knowing the rotational speed of the propeller, if we knew the direction of the threads of entry, it would not be difficult to determine the necessary conditions, unfortunately we are in the absolute ignorance of these directions, which are probably different at different points of the propeller. On the other hand, the reduced pitch of the helix, that seems to indicate no need for a low angle of attack, causes a large propulsive surface, consequently a large diameter, many arms, and requires a large number of turns of the propeller, otherwise the lift would be insufficient since it is expressed by $P = s.W^2.\alpha.\lambda$. All these conditions lead, in turn, the need to give the propeller in question a great resistance, because we are dealing with excessively considerable efforts due to centrifugal force; This weight is very considerable for the lifting propeller, which will absorb, for its own account, a considerable part of the lifting force. It will be, therefore, perhaps be more advantageous to increase the pitch, even if you use less power. Anyway, the issue of propeller lift seems to us very difficult to solve in practice and as we can not treat it from the theoretical point of view, we did not seek to bring it within the framework of this study.

To show the ease with which, thanks to our method, a propeller can be calculated for given conditions, we will, for example, calculate a normal propeller and a special propeller for two different cases.

Let us first assume the case of an airplane which, moves at a speed of 20 meters per second, which equals 72 kilometers per hour, would require the use of a 50 HP engine. This is, almost, the conditions of some current airplanes.

Let us say that the propeller is running at 600 rpm, or 10 revolutions per second. We will have:

$$F = 20,\ V = 20,\ \text{N}{=}10.$$

We need to determine the number of wings in the propeller. We will have, according to the equation of compatibility:

$$a = \frac{2500H.N^2}{V^5} = \frac{2500.50.10^2}{20^5} = 3.906\ .$$

So we can take a is equal to either 3 or 4. If we give three wings, it will increase the width of the wings by the reduction coefficient $q = \frac{3,906}{3}$; if, instead, we give 4 wings, this coefficient becomes $q = \frac{3.906}{4}$, which is close to unity. We will choose the number of 4 wings. As the equation of compatibility has shown us that we can adopt an acceptable number of normal wings, we will assume for the propeller normal wings. In the appendix, the elements of a normal wing, elements which are always the same for all the normal wings and all expressed according to the module. So we only have to determine the module of our wing.

The module M will be $\frac{A}{2\pi}$ or $\frac{V}{2\pi N}$ for this case:

$$M = \frac{20}{2.3.14.10} = 0^m.318\ .$$

So the radius of the hub will be:

$$r_0 = 0.5 \cdot 0^m.318,$$

which will give for the diameter of the hub:

$$d = 0^m.318,$$

and for the diameter of the helix:

$$D = 3^m.180\ .$$

The number of wings will be 4.

For the plot of the helix, we divide the radius into equal parts to the module, except the first division, which will be equal to half of the module:

Divisions du rayon ρ................	0^m159	0^m318	0^m636	0^m954	1^m272	1^m590
Valeurs de $\frac{H}{2\pi}$ M (tirées du tableau de l'appendice)..................	0^m345	0^m339	0^m344	0^m353	0^m362	0^m372
Valeurs du pas H =................ Les nombres précédents ont été multipliés par 2π.	2^m170	2^m132	2^m162	2^m216	2^m272	2^m334
Largeur spécifique constante.......	$L \times q = 0^m245$.					

In the table above we have arranged, in the first line, the divisions of the radius to which correspond the variable pitch of the wing, These pitches are arranged in the third line; they were obtained by multiplying, by 2π, the values of the preceding row, which are $\frac{H}{2\pi}M$, taken themselves from the table in the appendix. Lastly, in the last line, we have indicated the constant specific width of the wing, which is obtained by multiplying by M the width indicated in the table; moreover, we have multiplied it by the reduction coefficient q.

So all the elements of our propeller are determined enough to start construction.

As a second example, we will choose special conditions where normal wings could not be used.

Suppose a dirigible balloon propeller driven by an engine de 100 HP, moving at a speed of 14 meters per second, that is 50 kilometres per hour. Suppose moreover that the number of turns is 360 revolutions per minute, or 6 revolutions per second.

Let us see if it will be possible to give the thruster an acceptable number of normal wings; for this,

$$a = \frac{2500 \cdot 100 \cdot 6^2}{14^5} = 16.73\ .$$

As in reality, we will only be able to use 4 wings at the maximum, we must abandon the normal wings. The reduction coefficient will be:

$$q = \frac{16.73}{4} = 4.2 \text{ approximately.}$$

We saw above that if we gave the wing a radius equal to 7M, then the specific width, equal to $\frac{1}{6}$ of the length of the wing, increased the active area by 4.1 times. This is precisely what we have, appropriately, so we can take as a limit for the wing, $r_1 = 7M$.

To determine the modulus, we will ask:

$$M = \frac{V}{2\pi N} = \frac{14}{6.28 \cdot 6} = 0^m.371,$$

which will give us for the diameter of the hub:

$$d = 0^m.371,$$

and that of the propeller:

$$D = 14 \cdot M = 5^m.194,$$

the number of wings will be 4.

Divisions du rayon ρ..	0m185	0m371	0m742	1m113	1m484	1m855	2m126	2m597
Valeurs de $\frac{H}{2\pi}$.	0m403	0m592	0m401	0m411	0m422	0m433	0m444	0m456
Valeurs de H.........	2m525	2m480	2m522	2m576	2m645	2m715	2m785	2m862
Largeur spécifique constante.............	$L = 0{,}275\,M \cdot \frac{16{,}73}{4} = 0^m427.$							

In the table above, we have arranged, as before, the divisions of the radius by multiplying the module successively by the numbers 0.5, 1, 2, 3, 4, 5, 6 and 7. Below we have inscribed the variable pitch of this wing by multiplying the $\frac{H}{M}$ values, taken in the table of the appendix, by the module. Below, we have indicated the constant specific width which is, as we have seen above, equal to 0.275 of the module for $r_1 = 7M$, By multiplying this value by the reduction coefficient q, we find a width $0^m.427$ which is about one sixth of the length of the wing.

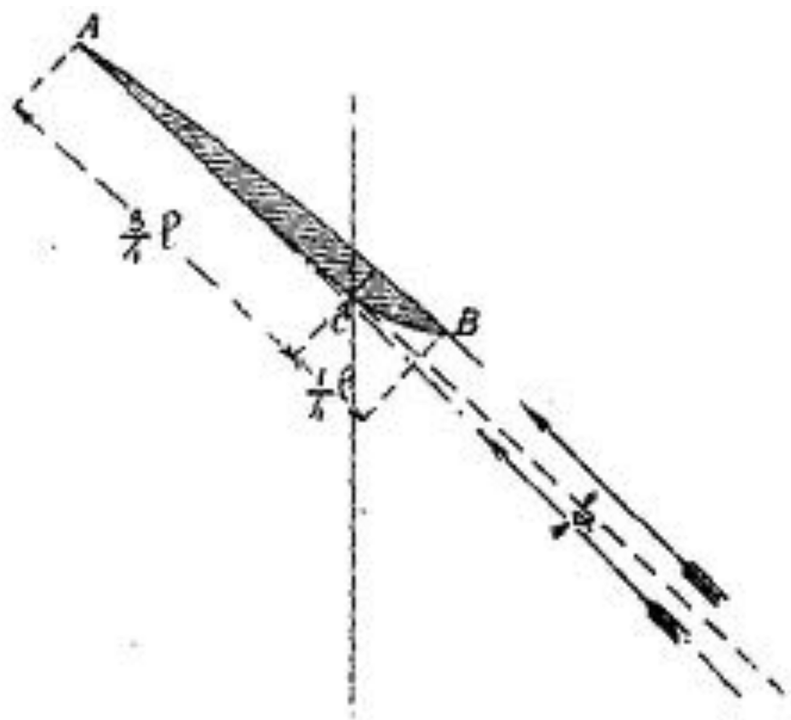
If we wanted to give the wing a shape other than a constant specific width, one would take the calculated figures by one of the formulas which we indicated above and one would multiply these figures by the module and by the reduction coefficient q.

Before finishing this study on aerial propellers, we still have some observations to make on the construction of the propeller wings, a construction which presents some special difficulties. These difficulties result mainly from the condition of very great lightness which the aerial propellers must satisfy by their very purpose; moreover, these difficulties are further increasing because of the large diameters and the number of considerable revolutions which it is necessary to give to the air thrusters; in these cases we have to deal with formidable efforts due to centrifugal force. For the wing to work in good conditions, it must, above all, not deform. Now, this deformation is inevitable for a thin wing when it is fixed on the arm of the propeller by the middle of its width, as is done ordinarily. Suddenly, there is a twisting due to the position of the centers of thrust of the air along the wing. When a thin plane attacks the air obliquely, the center of thrust is cut back to the anterior edge of the plane at a point which depends on the incidence; for very small impacts, like those we are dealing with for propellers, we can assume that this center of thrust is at a distance, from the entrance edge, equal to a quarter or fifth of the total depth of the thin plane. assuming for the wings of propellers a quarter of their width, we will be in conditions quite close to reality, so there will be all along the wing a line dividing the wing into two parts, one, before, one quarter of the total width and the other, to the rear, occupying three quarters of the width of the wing, this line will be the link of the centers of thrusts of the air. In order to avoid twisting the wing, it will be necessary to make the resistant arm, on which the surface of the wing is fixed, coincide. the wing, with this line of centers of thrust, For the wooden wings, this line will have to correspond to the greatest thickness of wood.

There is yet another condition to which the thruster wing must be subjected, it is that of not receiving harmful counter-pressure on its dorsal face; it is necessary, for that, to avoid that the air strings come to meet a part of this dorsal surface in a positive angle.

In order to satisfy these two conditions, a wing section similar to that shown in figure 11 can be adopted.

This figure, which is the corrected cylindrical section of a wooden wing, shows the active or pushing face of the wing in ACB. The line AC is straight and has been drawn by the process which we have indicated above, it makes, with the direction of the fluid threads, represented by the line MC, an angle of incidence close to 2°. The length AC is equivalent to three quarters of the total width of the wing AB, at this point of the radius; the length CB is only a quarter of this width. From the point C, corresponding to the thickest wing, the pushing face becomes convex, following a curve, as smooth as possible, which joins the entrance edge at a point B which is part of the dorsal side of the wing; in this way, the front part of the wing forms an entrance spout whose tip is on the dorsal face. This dorsal surface is constituted by a curve which, at the point B, is tangent to the direction of the fluid threads parallel to MC, and which joins the pushing face at the exit edge A, thus giving the wing as thin a section as possible in its back part. A wing built on this model would not have a tendency to bend, since the maximum thickness of the material is distributed along the line of the centers of thrust, and, moreover, it would receive on its dorsal surface no harmful counterpressure; on the other hand, it has the disadvantage of meeting the fluid threads under too high incidence, over the entire front part of the wing thanks to the curvature of the beak; This disadvantage would certainly lower the general coefficient of efficiency of the wing, but there is every reason to suppose that this lowering would be inferior to that which would be suffered by this same coeeficient if the wing were disposed so as to receive the fluid threads on the back part of the wing.



We can also adopt this same wing section, when the wing is not made of solid wood. It is possible to fix on the main beam, whether of wood or metal tube, transverse chords having the same section as that shown in figure 11, and spaced at a distance from each other along the radius, so as to form the helical surface of the wing. On these chords one would tend, on both sides of the wing, either a resistance fabric or even a thin sheet of metal; it would have a light wing, rigid and meeting the conditions of good operation.

If one wishes to build the wing with a thin metal blade, fixed on a rigid arm, one will begin by giving to the blade the shape of the calculated helical surface, then one will fix to it by one or several rows of rivets, or even by means of the autogenous welding, to the arm of the propeller which will form a rib on the dorsal surface of the wing; it is necessary to avoid naturally that the projection of this rib is not too pronounced.

As well as from a constructive point of view, the biplane has an advantage over the monoplane, so it seems to us possible to achieve good propulsive propellers by doubling the wings one behind the other, and placing them at a distance of about the width of a wing. This system, for thin metal wings, could have the advantage of increasing the active surface, while considerably increasing the solidity of the thruster; one could, in fact, connect the two parallel wings by transverse partitions curved along arcs of a radius corresponding to their position on the wing; these bulkheads would be spaced along the wing and triangulated by means of flat tie-rods working on the tension. Wings of this kind would be cellular wings and the transverse partitions would prevent air from flowing along the wing by channeling it in the transverse direction.
There is still a sort of propeller wing, little studied here, but which, in all probability, will be used in the future successfully, it is the flexible and elastic wing similar to the feathers of large birds. This wing could consist of a series of flat springs fixed, by their thickest end, along the arm of the propeller, which would also be flattened and form an entrance generator; these springs would be spaced at a distance from each other and form together a helical concave surface. The resistance of the springs to bending should be calculated so that the total thrust on the wing, divided by the number of springs, is sufficient to straighten the curvature of each spring, so as to give it a rectilinear direction doing with the direction of the fluid threads a very small angle. It would be necessary that the resistances of different springs are distributed in direct proportion to the

square of the corresponding radius, that is to say that the springs of the tip of the wing are much stronger than those of the base. The springs, once secured and tested separately by the dynamometer, would be covered with an elastic material which would be stretched at the moment of deformation of the springs. A wing of this kind would be difficult to construct, but we believe that, if it were properly calculated, it would achieve a propeller of very good performance.

We have reached the end of the task we have imposed on ourselves. In this study we have sought to to explain, as clearly as we could, a method which gives, on the question of helical thrusters, an overall view thanks to which it is possible for the investigator to see through the eyes of the mind the invisible phenomena that escape the physical view. This method is intended to enable the mind, guided by calculation and knowledge of the mechanical laws, to orient itself in the obscure maze of complex and little-known phenomena, or, for lack of a common thread, it loses the notion of necessary relations, no longer distinguishes them from contingencies and clings to the indications which he seizes, at random, in the observations of current practice, perhaps indicative in some cases, but, more often than not, seemingly misleading and distorting the reality of things.

For aviation, the perfection of thrusters is a question of the highest order, for the solution of which the empirical methods of groping are absolutely insufficient, it is therefore indispensable to illuminate it by the bright light that. alone, projects a general and scientific method.

Happy would we be if, in this work, we have succeeded in bringing to the question a contribution in this order of ideas.

Paris, 1909

S. DRZEWIECKI

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TABLEAU DES ÉLÉMENTS D'UNE AILE NORMALE

Subdivisions du rayon $\frac{\rho}{M}$	tang β	$\frac{H}{2\pi M} = \frac{\text{tang}\,\beta}{\text{tang}(\beta-\alpha)}$	Pas $\frac{H}{M}$	Largeur de l'aile $\frac{l}{M}$	Largeur spécifique constante $\frac{L}{M}$
Rayon du moyeu $\frac{r_0}{M}$	0,5	1,085	6,905	0,942	0,75
	1	1,066	6,685	0,969	»
	2	1,081	6,781	0,877	»
	3	1,108	6,945	0,791	»
	4	1,136	7,131	0,727	»
Rayon extrême $\frac{r_1}{M}$	5	1,167	7,318	0,677	»

$M = \frac{V}{2\pi N}$. Le nombre d'ailes $a = \frac{2500\ F.\ N^2}{V^2}$.

Les largeurs $\frac{l}{M}$ ont été calculées par la formule :

$$\frac{l}{M} = \frac{233}{2(z_1^3 - z_0^3) + 3 \cdot p\,(z_1^2 - z_0^2)} \cdot \frac{z'}{1 + pz}(z + p);\ p = 1$$

TABLEAU DES ÉLÉMENTS D'UNE AILE SUPÉRIEURE A LA NORMALE

Subdivisions du rayon $\frac{\rho}{M}$	tang β	$\frac{H}{2\pi M} = \frac{\text{tang}\,\beta}{\text{tang}(\beta-\alpha)}$	Pas $\frac{H}{M}$	Largeur spécifique constante $\frac{L}{M}$	Rapport $\frac{L}{r_1 - r_0}$	a' nombre d'ailes réel a nombre d'ailes calculé
Rayon du moyeu $\frac{r_0}{M}$.	0,5	1,085	6,905			$a = \frac{2500\ FN^2}{V^2}$
	1	1,066	6,685			
	2	1,081	6,781			
	3	1,108	6,945			$q = \frac{a'}{a}$
Rayons extrêmes $\frac{r}{M}$ . .	4	1,136	7,131	Jusqu'à $r_1 = 5M$	$\frac{1}{6}$	
	5	1,167	7,318	0,750		Il y aura lieu
	5,5	1,182	7,415	Jusqu'à $r_1 = 6M$	$\frac{1}{13}$	de multiplier
	6	1,197	7,508	0,427		les largeurs $\frac{M}{L}$
	6,5	1,214	7,615	Jusqu'à $r_1 = 7M$	$\frac{1}{25}$	
	7	1,230	7,713	0,275		par le rap-
	7,5	1,244	7.800	Jusqu'à $r_1 = 8M$	$\frac{1}{44}$	port q.
	8	1,262	7,916	0,173		