Gearing for Gearheads

Part 3

by Phillip Miller

Part 1 presented the basic geometry and dimensions for several Rolls-Royce Merlin Propeller Speed Reduction Unit (PSRU) gearsets. Part 2 continued with derivations of defining formulae of tooth beam equivalent dimensions, tooth stresses, rub velocity, pitch line velocity and contact ratio. Part 3 will examine how gears are made, describe the materials and processes involved, complete the analytical tools needed to evaluate gearsets, evaluate the relative merits of the gearsets already discussed, and present observations and conclusions.

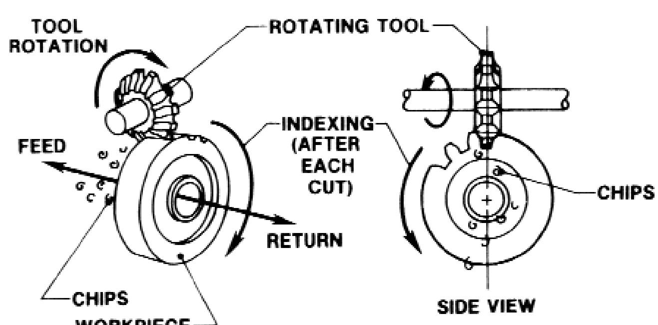
Gear Manufacture

Shaping the Gear Blank

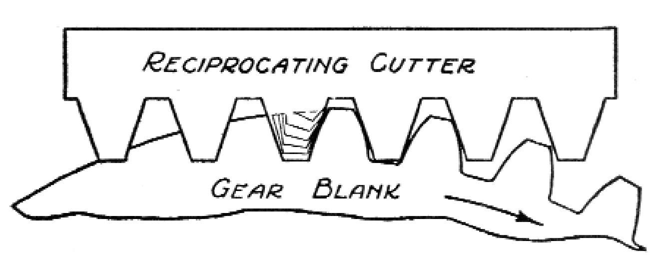
During the Merlin's era, four spur gear production processes were already in wide use: formed-tooth milling, straight cutter generation, rotary cutter generation and hob cutter generation. Two of these four (rotary and hob cutter generation), followed by form grinding were apparently used in Merlin PSRU gear production.

The formed-tooth process, oldest of the four, used a formed milling cutter to remove material between gear teeth. The gear blank was then indexed to the next tooth space and another milling pass made. This process required skilled machine operators and was time consuming. A variant of this, which used a formed grinding wheel, seems to have been used for final form grinding of Merlin gear teeth.

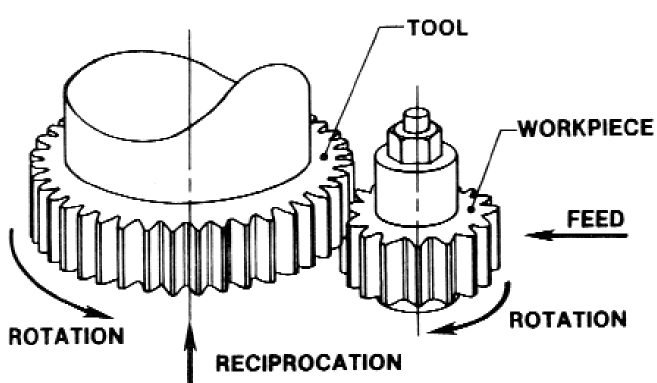
Straight cutter generation exploits the fact that any involute gear of a given pitch will mesh with a rack of the same pitch. The rack may be used as a cutter if it is attached to a reciprocating ram. This is similar to a horizontal shaper, except that with the gear shaper the ram axis is typically vertical. When the cutter and gear blank are clear of one another after each cutting stroke the rack advances along its length and the gear blank rotates slightly. Since the rack is not infinite in length it must return to be re-indexed and restarted at intervals. straight rack gear shapers must be large to accommodate the long racks.



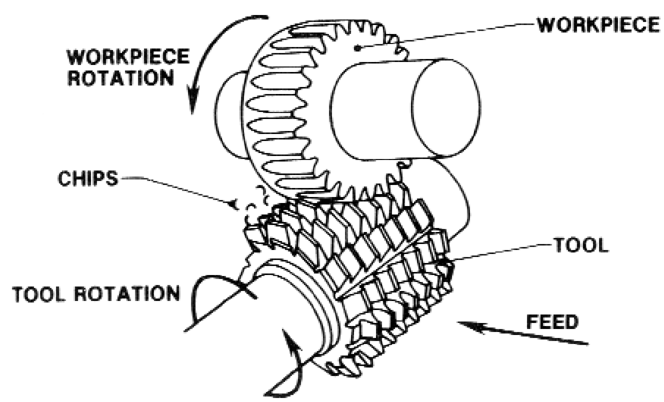
FORMED TOOTH



STRAIGHT CUTTER



ROTARY CUTTER



HOB CUTTER

Four production gear shaping techniques were in use when Merlins were built.
(Formed Tooth, Rotary Cutter and Hob Cutter illustrations courtesy of Industrial Press, Inc.)

Rotary cutter generation makes use of a circular cutter to avoid the time consuming stopping and re-indexing. Both cutter and gear blank advance incrementally when the cutter clears the gear blank during each return stroke of the ram. The Fellows gear shaper is a tool which affects this generation technique.

Hob cutter generation, or hobbing, employs a shaping tool called a "hob" that resembles a helical thread tap. Both the hob and the gear blank revolve constantly as the hob is fed across the face width of the gear blank. The hob rotation axis is skewed so that it presents its cutting teeth along the rotating axis of the gear blank.

Shaping Considerations

Small diameter gears with relatively large and strong teeth (e.g., PSRU pinions) are often subject to "undercutting" of the tooth involute surface in the vicinity of the base circle during machining with any of the usual "generating" type tooling. This weakens the tooth and may also reduce the contact ratio. Undercutting occurs when the tips of the generating tool cutters overlap the tangential intersection of the contact line and the base circle of the gear being cut. This can be avoided by limiting the use of "generating" tool machining. "Rough" hobbing may be used to create the initial shape, followed by form grinding to final size and shape, thereby avoiding undercutting.

The Fellows Gear Shaper

The Fellows gear shaper was a labor and time saving improvement over the rack shaper. It was essentially the work of one man, Edwin R. Fellows, born in 1865 and with an initial career as a window dresser. At the age of 24 he found employment as a draftsman for a machine tool company and became interested in gearing and gear production. He worked at night to find a way around the shortcomings of the rack shaper and eventually arrived at the Fellows gear shaper. This solution, however, required a generating and grinding method for production of the circular gear-shaped cutting tools. The Fellows Gear Shaper Co. was formed in 1896 and delivered its first machine in 1897. Edwin R. Fellows received the Franklin Institute John Scott Medal for his technical accomplishments in 1899.

While the Fellows gear shaper provided a large improvement in its day, it was eventually supplanted by hobbing for most serious production. BUT the grinding tooling and methods Edwin R. Fellows developed to produce hard, sharp and precise cutting tools for his Fellows gear shaper served to grind hardened gears as well. This is considered a greater contribution to the "Gearhead" than the Fellows Gear Shaper. The R-R Merlin owes a great deal of its success to the use of hardened and ground gears and the production of these no doubt benefited from Fellows grinder development even if the lineage of particular production machinery does not show a direct link.

Where small pinions drive much larger gears (as in the Merlin PSRU), the strength of pinion teeth can be increased by making them thicker at the expense of the gear teeth. This is accomplished by starting with a slightly larger pinion blank and offsetting the shaper cutter or hob away from the pinion center. This process changes the tooth thickness without changing the pitch diameter. A corresponding change must be made when machining the gear by starting with a smaller blank and offsetting the tool toward the gear blank center. This operation causes a shifting of addendum and dedendum (or root diameter) and hence one of its popular names—shifted addendum. It appears that the 0.420:1 ratio Merlin gearing involves this sort of shifty manipulation. The earlier 0.477:1 gearset exhibits evidence of tooth thickening and thinning of a more elegant nature. It is possible to provide tooling that will directly (without offset) generate teeth of desired thickness. Thus a thickened tooth shaper or hob will directly provide narrowed tooth gears while a narrowed tooth shaper or hob will result in thickened tooth pinions all without changes to addendum, dedendum (or root diameter) and OD of either gear or pinion. The dimensions on R-R drawings D.20685/1 and D.20684/1 (65-tooth 0.477:1 ratio) suggest this option.

Materials and Processing

Gear materials must be initially soft enough to shape by one of the methods previously described. However, in service it must be hard enough to resist wear and breakage. These conflicting objectives required gears to be made from materials that can be hardened after they have been shaped.

The three R-R gear drawings call out "S/ZNC" material, with a core Brinell Hardness Numbers of 388 to 444 and a case hardness value of Rockwell RC-57 to RC-65. The core hardness translates to a tensile strength of approximately 190-225 ksi and the case hardness translates to approximately 295-335 ksi equivalent in the presumably somewhat brittle hard case wearing surface which is called out as 0.045" to 0.055" deep.

"Double quenching" is called out on all the drawings and this presumably involves double heating as well. We assume this aids in effective conversion of the carbon rich case to martensite. The pinion drawing calls out a 5 hr at 160°C drawing cycle and the driven gear drawings call out 2-4 hr at 125-140°C cycle, both after the case hardening carbon infusion cycle. There is no mention of a near carburizing temperature normalizing cycle after case hardening. The indicated tensile strengths and hardness values are higher than common for more mundane applications and this is likely the result of “fine tuning” of either (or more likely both) the “S/ZNC” alloy and the heat treat procedure. All three drawings call out a 0.010″ minimum grind after heat treat for cementite removal. As we’ll see later, this is significant.

Thorough perusal of the Merlin contemporary (1940) *Machinery’s Handbook* left the impression that “S/ZNC” alloy may be somewhat like SAE 4615 (a nickel-manganese-molybdenum carburizing alloy) or SAE 3312 (a nickel-manganese-chromium carburizing alloy). I suspect that it may be a nickel-manganese-chromium-molybdenum alloy since that is a known good combination that is notably scarce where raw economics (prevalent with the Great Depression on both sides of the Atlantic) abounded. Whatever the alloy or its designation, one intended for case hardening usually differs from its high strength brethren by having a slight initial carbon content in anticipation of heavy carbon infusion from the surface during the carbonization phase of case hardening. The chromium-nickel-molybdenum American Iron & Steel Institute AISI 8620 carburizing steel appeared in civilian tech literature after WWII and was likely a related or at least a similar product with some wartime development.

Evaluation Tools

The Modified Lewis Formula

The critical influence of filet radii upon stress concentration is often obvious but the means of analyzing its influence developed somewhat after the pre-1900 debut of the Lewis formula. The

MODIFIED LEWIS EQUATION

A factor β, added to the basic Lewis equation, incorporates the influence of tooth base fillet radii (r_f). Thus we have:

$$S_b = \frac{F_b}{\beta Y p} \quad \text{where} \quad \beta = \frac{1.6}{\left(1 + (0.075\,p)/r_f\right)} \quad \text{or}$$

$$S_b = \frac{F_b\left(1 + (0.075\,p)/r_f\right)}{1.6\,\beta\,Y\,p}$$

which is the equivalent bending stress (in psi)

Fig. 1. The Modified Lewis Equation, derived from a presentation by Spotts, attributed to the work of Timoshenko and Baud, circa 1916.

Lewis formula, modified to account for tooth filet radius as a variable, was later developed by incorporating the 1916 work of Timoshenko and Baud. This added a factor β to the original Lewis equation and an additional formula (fig. 1) was devised to calculate β. Wilfred Lewis remained active in his correspondence with other prominent “Gearheads” including Daniel Adamson of The British Institution Of Mechanical Engineers. Lewis proposed, in a 1923 paper presented to the ASME, the design and construction of a gear test machine capable of measuring dynamic increment loads. According to Earle Buckingham (introduced in Part 1), the Lewis gear testing machine was built and tests run at MIT over several years resulting in the 1931 ASME Research Paper “Dynamic Loads On Gear Teeth”.

Surface Stress and Dynamic Loads

Direct data does not exist for analyzing Merlin gear tooth surfaces in contact with one another. However, with some backtracking and detective work, satisfactory estimates can be calculated.

Early in the 20th century, Earle Buckingham and others applied Heinrich Hertz’s stress equation to the problem of involute gear surfaces transmitting tooth to tooth loads. This resulted in

Heinrich Rupert Hertz was a late nineteenth century German Physicist whose analysis, experiments and skillfully fabricated apparatus led to demonstration of high frequency electromagnetic wave generation, short range transmission and verified reception. A modest man, he told his students that it wasn’t likely to be very useful! After his death, Hertz’s work inspired Guglielmo Marconi to demonstrate the first transatlantic transmission and reception of radio frequency (RF) energy, a precursor to what would be known as “radio”. We now speak of frequency in “Hertz” instead of cycles per second just to honor and remember Heinrich Rupert.

That’s fine for electrical engineering types and the USMC did send me through USN Electronics School but I’m really a mechanical engineer at heart. My hero is the Heinrich Rupert Hertz who wrote the equations and developed the elliptical integral solutions for the stress/strain model of curved elastic bodies in forced contact. This work was essential to the understanding, analysis and development of not only involute gears but also rolling element bearings, cams and followers.

Hertz biography web sites don’t typically acknowledge this stresser’s breakthrough and mechanical engineers generally wouldn’t know RF if it bit them on the nose. Hence it seems nearly impossible to find one site that shows that these two miraculous achievements are attributable to the same man. Yet, all peripheral data points to the fact. But with the Hertz stress as well as the radio achievements, many others carried on and made careers from continuation of his work after the too-early death of Heinrich Rupert Hertz. Thus we work in these fields from the works of others with scarcely any knowledge of the real pioneer.

Earle Buckingham's 1926 paper "Limiting Loads for Wear on Spur Gears" published by the American Gear Manufacturers Association. Five year later the American Society of Mechanical Engineers special gear committee was formed. Its work including a test program directed by Earle Buckingham to determine sustainable surface compression stresses for a variety of material combinations under rolling and sliding contact simulating the action of involute gears.

Buckingham applied the Hertz equation to transform the load test results into more useful Hertzian stress values, initially presenting a version of the Hertz equation for cylindrical surfaces, which he later developed into an equation readily applicable to meshed spur gears. He also included equations for calculating the depth and value of maximum shear stress below loaded involute gear surfaces. The latter are particularly useful but again the more user-friendly presentation in Spotts is handier for dealing with transmitted load, and sustainable Hertzian stress values in actual gear examples (fig. 2). It seems worth noting that, to the everlasting gratitude of very mediocre mathematicians like me, the presentations of Buckingham, et al, have eliminated the elliptical integrals of the original Hertz solution. Some curve fitting simplification is suspected but at least the results are now readily useful.

Rockwell Hardness tests became popular and to some extent supplanted Brinell hardness tests in the 1930s so translation from one to the other is often necessary when dealing with technical literature from pre-WWII years. The Brinell to Rockwell conversion chart from the 1940 10th Edition *Machinery's Handbook*, the Buckingham table of sustainable compressive stress as presented by Spotts, together with the R-R drawing callout of case hardness of RC-57 to RC-65 shows that the 0.420:1 ratio 71 tooth gearset of the Merlin 724 civil transport engine of 1955 should handle a surface compressive stress of 240.4 ksi at the drawing median hardness of RC-61. This and the considered assumption that the Merlin 724 was displaying satisfactory PSRU life while delivering a cruise output of approximately 1,200 hp @ 2,650 rpm allow us to calculate a minimum (at RC-57) load limit for wear for pitch circle contact as most surface fatigue failures occur at this location.

Figure 3 uses the compressive stress limits at various hardness callouts and other known 0.420:1 gear and Merlin 724 engine details. These values are then inserted in the Spotts' version of the Buckingham dynamic load equation (fig. 4). We now solve for the rather involved "constant C", whose value will be used to determine the relative anguish caused by modified unlimited race

DEPTH TO POINT OF MAXIMUM SHEER UNDER HERTZIAN LOADING

$$Z = 1.19\left[\frac{F_t\left(r_1 \times r_2\right)}{F\ E\left(r_1 + r_2\right)}\right]^{1/2}$$

where Z = depth of maximum sheer stress

F_t = tooth contact load

F = tooth face width

E = material modulus

$r_1 = r_{p1}\sin\theta$, r_{p1} = pitch radius of gear 1

$r_2 = r_{p2}\sin\theta$, r_{p2} = pitch radius of gear 2

θ = pressure (contact) angle

MAXIMUM SHEER STRESS

$S_s \cong 0.304\ S_c$

where S_s = maximum sheer stress

S_c = maximum compressive stress

Fig. 2. Depth to Point of Maximum Sheer under Hertzian Loading, as presented by Buckingham. Notation modified to agree with other examples herein.

LOAD LIMIT FOR WEAR AND PITTING

$Fw = Dp\ F\ Q\ K$

Where

Fw = pitch circle load limit for wear and pitting

Dp = pinion pitch diameter (5.1111")

F = tooth face width (2.625")

$$Q = \frac{2N_G}{N_G + N_P} = \frac{2 \times 50}{50 + 21} = 1.408$$

And: $$K = \frac{(Sc)c^2 \sin\theta}{1.4}\left(\frac{1}{E_1} + \frac{1}{E_2}\right)$$

Where

$\theta = 25°$, $E_1 = E_2 = 30 \times 10^6$ (steel modulus),

$(Sc)c$ = compressive endurance stress

Interpolation and Extrapolation for RC-57 to RC-65

$$\frac{(230 - 190)ksi}{(59 - 51.3)RC} = \frac{40ksi}{7.7RC} = 5.2 \text{ ksi/RC unit}$$

$\therefore$ $(Sc)c$ @ RC-57 = 219.6 ksi, Fw = 25666.6

and $(Sc)c$ @ RC-61 = 240.4 ksi, Fw = 30759.0

where RC-61 is a median value

Fig. 3. Load Limit for Wear and Pitting, as presented by Spotts, based on Buckingham's work with the Hertz equation, 1926 and thereafter.

engines at typically increased rpm and to explore the implication of modified gear ratios. The "C" values show the R-R Merlin PSRU to be quite tolerant of the expected dynamic loading. The torsionally flexible quill drive, lube system and accurately finished hardened gears in a compact and fairly rigid case deserve the credit.

Evaluation

Table 1 presents results for several gearsets operating at both stock and racing power levels. Results for "modern" tooth forms at racing power levels are also included for comparison. At the left of Table 1 are the gearset identifiers and independent variables that define each one. On the right are calculated "measures of goodness", each of which is summarized below. **Gearset identifiers** (such as **R-R 71 @ 0.420 Stock**) and **measures of goodness** are set in **bold type** for the following discussion of Table 1.

Modified Lewis Y — bigger is better.
Contact Ratio— typically between 1.4 and 1.7. Bigger is better, but sliding velocities increase with contact ratio.
Pitch Line Velocity — values below 5,000 fpm do not usually introduce special lubrication considerations.
Sliding Velocities — two values are presented: Gear Sliding over Pinion and Pinion Sliding over Gear. These are typically 25%to 35% of the pitch line velocity. Smaller is better.
Dynamic Load — As described above, this is derived from the Merlin 724 gearsets used in transport service (**R-R 71 @ 0.420 Stock**). The value presented is for a pinion tooth surface hardness of RC-57, the minimum specified by R-R gear drawings. Lower is better, but even values slightly above 50,000 psi are not unreasonable.
Max Bending Stress — Lower is better. Values over 95 ksi (half of the tooth material core hardness of 190-225 ksi) have proved troublesome.

As one would expect, the **R-R 65 @ 0.477 Stock** gearset handles the stock engine output well with no measures of goodness out of line. The **Max Bending Stress** is approximately 55% of our limit and **Dynamic Load** is lower than that "proven" with the Merlin 724 gearset, which develops a higher contact line load.

The "cushion" naturally diminishes when this gearset is subjected to the output of a race prepared Merlin. The bending stress rises to 94% of our limit and the calculated dynamic load range rises considerably above that of the Merlin 724. However, experience does not show chronic surface pitting, wear or failure with unlimited racers so equipped. This indicates that dynamic loads of at least this order should be satisfactory for modified gearsets in unlimited racers.

The bottom grouping is composed of more or less "standard" gearsets of past and present origin. Comparison of the **R-R 65 @ 0.477 Race** thickened 20°tooth form with the "standard" **S 20° 65 @ 0.477 Race** tooth form is enlightening. The **Modified Lewis "Y"** column shows a 40% advantage to the R-R initial choice. Pound for pound this early 1930s Merlin gearing was outstanding. But wait—the **S 25° 65 @ 0.477 Race** gearset in the bottom grouping is a harbinger of thing to come. Comparison of the "standard" 20° pinion (**S 20° 65 @ 0.477 Race**) with the "standard" 25° version (**S 25° 65 @ 0.477 Race**) shows a 30% **Modified Lewis "Y"** advantage to the 25° pinion. Note that there are no thicker than nominal teeth in either pinion of this example.

The 25° contact angle involute tooth form was not unknown in the 1930s. However, there was little or no applicable service experience, no handy preexisting tables of 25° involute Lewis Y factors, and no electronic calculators or spreadsheets, making comparison of gearsets a difficult, time consuming and serious undertaking. It involved arduous large scale drafting followed by somewhat arbitrary scaling measurement.

EQUATION FOR DYNAMIC LOAD

$$Fd = Ft + \frac{0.05V(FC + F_T)}{0.05V\,(FC + F_T)^{1/2}}$$

where Fd = dynamic load (lbs),
Ft = pitch circle tangent load
F = tooth face width
C = a factor involving rotating masses, tooth form and tooth form dimensional error

$$Ft = \frac{33000\,hp}{V}$$

$$V = \frac{\pi\, D_P \times rpm}{12}$$

where V = pitch line velocity,
Dp = pinion pitch diameter.
Let $Fd = Fw$, $Dp = 5.1111$, $rpm = 2650$, $hp = 1200$
Then solving by iteration,
$C = 6505$ @ RC-57

Fig. 4. ASME/Buckingham Equation for Dynamic Load (circa 1931) as presented by Spotts.

Table 1. Characteristics of Several Gearsets

Gearset Identifier	PN	GN	*phi*	HP	RPM	C	F	t_O	Pr_f	Pa	Pb	Ga	Gb	N	m_G	P	LewisYm	CR	V	Vs_P	Vs_G	F_d_57	S
Manufacturer (S – Standard) Tooth @ Ratio Stock or Race Power	Pinion Teeth	Gear Teeth	Pressure Angle (degrees)	Power (hp)	Revolutions per Minute	Center Distance (inches)	Face Width (in)	Pinion Tooth Width Over Nominal (inches)	Pinion Fillet Radius (inches)	Pinion Addendum (inches)	Pinion Dedundum (inches)	Gear Addendum (inches)	Gear Dedundum (inches)	Gearset Teeth	Gear Ratio	Diametral Pitch	Modified Lewis "Y"	Contact Ratio	Pitch Line Velocity (ft/min)	Velocity of Gear Sliding over Pinion (ft/min)	Velocity of Pinion Sliding Over Gear (ft/min)	Dynamic Load at RC-57 (lbs)	Max Bending Stress (psi)
"Stock" Power Ratings																							
R-R 65 @ 0.477 Stock	21	44	20	1760	3000	8.640	2.625	0.0130	0.125	0.2658	0.3323	0.2658	0.3323	65	0.477	3.7616	0.106781	1.65	4385	-1477	1577	30145	52649
R-R 71 @ 0.420 Stock	21	50	25	1760	3000	8.640	2.625	0.0082	0.100	0.2434	0.3042	0.2434	0.3042	71	0.420	4.1088	0.125305	1.48	4014	-1123	1187	31204	53369
Mod R-R 65 @ 0.354	17	48	20	1760	3000	8.640	2.625	0.0130	0.125	0.2658	0.3323	0.2658	0.3323	65	0.354	3.7616	0.095539	1.63	3550	-1315	1459	32832	71572
Mod R-R 71 @ 0.340	18	53	25	1760	3000	8.640	2.625	0.0082	0.100	0.2434	0.3042	0.2434	0.3042	71	0.340	4.1088	0.113472	1.47	3441	-1043	1124	33275	67949
Allison 70 @ 0.556 Stock	25	45	25	1061	2600	10.000	2.563	0.0000	0.115	0.2857	0.3571	0.2857	0.3571	70	0.556	3.5000	0.126205	1.49	4862	-1270	1314	21798	23255
Allison 66 @ 0.500 Stock	22	44	25	1592	3000	8.250	2.563	0.0000	0.115	0.2500	0.3125	0.2500	0.3125	66	0.500	4.0000	0.126827	1.47	4320	-1223	1277	28305	44292
Allison 74 @ 0.424 Stock	22	52	25	1830	3400	9.250	2.563	0.0000	0.115	0.2500	0.3125	0.2500	0.3125	74	0.423	4.0000	0.126821	1.48	4896	-1317	1389	29441	44927
"Race" Power Ratings																							
R-R 65 @ 0.477 Race	21	44	20	3400	3400	8.640	2.625	0.0130	0.125	0.2658	0.3323	0.2658	0.3323	65	0.477	3.7616	0.106781	1.65	4969	-1674	1787	44591	89742
R-R 71 @ 0.420 Race	21	50	25	3400	3400	8.640	2.625	0.0082	0.100	0.2434	0.3042	0.2434	0.3042	71	0.420	4.1088	0.121470	1.48	4549	-1273	1346	46652	93842
Mod R-R 65 @ 0.354 Race	17	48	20	3400	3400	8.640	2.625	0.0130	0.125	0.2658	0.3323	0.2658	0.3323	65	0.354	3.7616	0.095539	1.63	4023	-1491	1653	49781	121998
Mod R-R 71 @ 0.340 Race	18	53	25	3400	3400	8.640	2.625	0.0082	0.100	0.2434	0.3042	0.2434	0.3042	71	0.340	4.1088	0.110191	1.47	3899	-1182	1274	50624	119272
Mod R-R 65 @ 0.383 Race	18	47	20	3400	3400	8.640	2.625	0.0130	0.125	0.2658	0.3323	0.2658	0.3323	65	0.383	3.7616	0.098483	1.64	4259	-1535	1685	48287	112279
Mod R-R 83 @ 0.339 Race	21	62	25	3400	3400	10.100	2.625	0.0130	0.125	0.2434	0.3042	0.2434	0.3042	83	0.339	4.1088	0.138157	1.49	4549	-1200	1286	46652	82557
Allison 70 @ 0.556 Race	25	45	25	3400	3400	10.000	2.563	0.0000	0.115	0.2857	0.3571	0.2857	0.3571	70	0.556	3.5000	0.126205	1.49	6358	-1661	1718	39331	56988
Allison 66 @ 0.500 Race	22	44	25	3400	3400	8.250	2.563	0.0000	0.115	0.2500	0.3125	0.2500	0.3125	66	0.500	4.0000	0.126827	1.47	4896	-1386	1447	44757	83466
Allison 74 @ 0.424 Race	22	52	25	3400	3400	9.250	2.563	0.0000	0.115	0.2500	0.3125	0.2500	0.3125	74	0.423	4.0000	0.126821	1.48	4896	-1317	1389	44757	83470
The Following Use "Modern" Tooth Forms whose Fillet Radii are Functions of Diametral Pitch. All are "Race" power ratings																							
S 20° 65 @ 0.477 Race	21	44	20	3400	3400	8.640	2.625	0.0000	0.080	0.2658	0.3323	0.2658	0.3323	65	0.477	3.7616	0.075882	1.65	4969	-1674	1787	44591	126122
S 25° 65 @ 0.477 Race	21	44	25	3400	3400	8.640	2.625	0.0000	0.080	0.2658	0.3323	0.2658	0.3323	65	0.477	3.7616	0.098883	1.47	4969	-1445	1515	44591	96517
S 20° 71 @ 0.420 Race	21	50	20	3400	3400	8.640	2.625	0.0000	0.073	0.2434	0.3042	0.2434	0.3042	71	0.420	4.1088	0.075882	1.66	4549	-1475	1594	46652	150481
S 25° 71 @ 0.420 Race	21	50	25	3400	3400	8.640	2.625	0.0000	0.073	0.2434	0.3042	0.2434	0.3042	71	0.420	4.1088	0.098883	1.48	4549	-1273	1346	46652	115158
S 20° 65 @ 0.354 Race	17	48	20	3400	3400	8.640	2.625	0.0000	0.080	0.2658	0.3323	0.2658	0.3323	65	0.354	3.7616	0.067449	1.63	4023	-1491	1653	49781	172511
S 25° 65 @ 0.354 Race	17	48	25	3400	3400	8.640	2.625	0.0000	0.080	0.2658	0.3323	0.2658	0.3323	65	0.354	3.7616	0.085034	1.46	4023	-1296	1398	49781	136314
S 20° 71 @ 0.340 Race	18	53	20	3400	3400	8.640	2.625	0.0000	0.073	0.2434	0.3042	0.2434	0.3042	71	0.340	4.1088	0.069653	1.65	3899	-1362	1512	50624	189112
S 25° 71 @ 0.340 Race	18	53	25	3400	3400	8.640	2.625	0.0000	0.073	0.2434	0.3042	0.2434	0.3042	71	0.340	4.1088	0.088944	1.47	3899	-1182	1274	50624	147577
S 20° 65 @ 0.383 Race	18	47	20	3400	3400	8.640	2.625	0.0000	0.080	0.2658	0.3323	0.2658	0.3323	65	0.383	3.7616	0.069653	1.64	4259	-1535	1685	48287	158500
S 25° 65 @ 0.383 Race	18	47	25	3400	3400	8.640	2.625	0.0000	0.080	0.2658	0.3323	0.2658	0.3323	65	0.383	3.7616	0.088944	1.46	4259	-1333	1425	48287	123688
S 25° 70 @ 0.556 Race	25	45	25	3400	3400	10.000	2.563	0.0000	0.086	0.2857	0.3571	0.2857	0.3571	70	0.556	3.5000	0.109309	1.49	6358	-1661	1718	39331	65797
S 25° 66 @ 0.500 Race	22	44	25	3400	3400	8.250	2.563	0.0000	0.075	0.2500	0.3125	0.2500	0.3125	66	0.500	4.0000	0.101938	1.47	4896	-1386	1447	44757	103845
S 25° 74 @ 0.424 Race	22	52	25	3400	3400	9.250	2.563	0.0000	0.075	0.2500	0.3125	0.2500	0.3125	74	0.423	4.0000	0.101938	1.48	4896	-1317	1389	44757	103845

NOTES:

1. Unless otherwise indicated, all varibles refer to pinion values. Where confusion might exist, a leading "P" indicates pinion values, whereas a leading "G" indicates gear value
2. All angles are in radians
3. Variable nomenclature: Read Pr_f (Pinion Filet Radius) as "pinion r sub f", PD_p (Pinion Pitch Diameter) as "pinion D sub p", etc

Fortunately, R-R persisted and discovered the benefits to design the 71-tooth, 25°, 0.420:1 ratio gearset. The earliest date I find on the drawing available to us (that of the 50 tooth driven gear) is 8-11-43. A mild thickening of the pinion teeth (0.0082″) was inferred by PSRU backlash clearance specification and the driven gear drawing pitch radius tooth thickness dimension. Tooth bending stress with **R-R 71 @ 0.420 Race**, 25° gearset in a race prepared Merlin is higher than with the **R-R 65 @ 0.477 Race**, 20° thicker-toothed gearset, partially because of a higher contact line load resulting from a smaller diameter though identical tooth count pinion. The dynamic load range with a race prepared Merlin is also slightly higher than with the 0.477:1 ratio gearset, but neither tooth bending stress nor dynamic load failure appear troublesome with the 0.420:1 ratio stock gears in unlimited racer use.

It seems appropriate at this point to consider important details of the unlimited racer experience. First, it is unlikely that all race prepared Merlins are equal in performance and it is doubtful if performance with a given gearset is often estimated from more than the rpm, boost pressure, and in the past, nitrous oxide flow. This certainly suggests possible scatter in the test results. Second, the time accumulated at "full power", which we have standardized for convenience as 3,400 hp @ 3,400 rpm, is minimal when compared to even military use. Do any of the top unlimited racers accumulate as much as five hours annually at "full power"? This is not intended as criticism, but rather as possible guidance in determining acceptable limits for contemplated custom ratio gearsets.

At least two custom Merlin gearsets have been tried in unlimited racing. One (**Mod R-R 65 @ 0.383 Race**) was a 65-tooth, 20°, 0.383:1 ratio gearset with the pinion tooth thickness assumed 0.013″ over nominal, which was reported as successful . The other (**Mod R-R 71 @ 0.340 Race**) was a 71-tooth, 25°, 0.340:1 ratio gearset with pinion tooth thickness assumed 0.0082″ over nominal, which reportedly failed. The failed 0.340:1 ratio set shows higher maximum bending stress and dynamic load.

It is tempting to draw conclusions and guidance from this comparison but it is of doubtful wisdom in view of the uncertainties of our assumptions, the statistics (only two test samples!) and the assumed identical versus actual outputs of the Merlins. Additionally, the racers favor ratios lower than 0.383:1. Attempts to achieve this with proven tooth forms, the Merlin PSRU center to center distance and gear face widths yield the even more discouraging stress and dynamic load results (see **Mod R-R 65 @ 0.354 Race**).

Observations and Conclusions

A Real Bottom Line

The "bottom line" (apologies to any MBAs who happen to read this) is that lots more horsepower at a slightly higher rpm is best handled by LARGER GEARS. Now at first reckoning I lumped thicker gear teeth into this category. I then manipulated some of the applicable derivations to solve for the tooth thickness required to bring the bending stress level of the 71-tooth, 25°, 0.340:1 ratio failed gearset down to the same level as a stock 71-tooth, 25°, 0.420:1 ratio gearset running in race prepared Merlins. The preliminary result was not as outlandish as feared. A further pinion tooth thickness increase (these teeth were already assumed 0.0082″ thicker than nominal) of roughly 11% was indicated. This amounted to 0.0429″ and would require that the 0.100 fillet radius be reduced to 0.0785″ with the original "inter fillet quadrant" tolerance flat remaining. Teeth of the driven gear would require thinning by a similar amount UNLESS the heretofore unquestioned 0.025″ running clearance could be safely (for limited unlimited racing use) reduced.

The fillet radius reduction of course reduces the modified Lewis "Y" factor and this has not yet been explored, but should be. The total effect of tooth thickening, in turn, could be optimized by iteration. There are other concerns to address before anyone should seriously consider this approach: Driven gear tooth strength? Interference at the meshed pinion OD? Heat treat complications with the thinner driven gear teeth, etc., etc? This concept does, however, present the POSSIBILITY of a successful very favorable ratio gearset shy of SERIOUS changes in gear types and/or PSRU dimensions. It is presented in that light with the hope that interested parties will further explore it.

A Fishing Expedition

A change to helical spur gears was briefly explored, but even with a mere 15° off-axis helix angle the gear-generated thrust can grossly overbalance the prop thrust at full power and 500 mph. The thought of in-flight transients added to this steady state quickly cancelled hope and interest in "non herringbone" helical spur gears.

This leaves us to acknowledge (and little more) that a set of "herringbone" gears could do the job. Helical gears of 30°-45° "off-axis" helix inclination vastly increase the effective working contact ratio, so can handle high loading with a decrease in tooth bending stress as well as dynamic load.

The real challenge to the "herringbone" concept is with the creation of such gears within the limited axial dimensions of existing Merlin PSRU cases. Beyond that, the problem of finding a vendor willing to tackle such a problem for a very low volume application is daunting. Good luck!

Increased Gearset Face Width

Tooth and gear strength will nominally increase linearly as gear face width increases. The PSRU cases must be extended with either a new and "longer" front case or an extension ring bolted between the existing PSRU front and rear cases. Wider-faced gears will be required but should be the least of the problems in this approach. There is opportunity in this modification to re-configure the longer driving pinion to receive its torque input at its center (stock Merlin pinions are driven from the forward end) from an internal, concentric, thick-walled sleeve, driven in turn by a longer, smaller diameter and thicker-walled torsional quill of appropriate flexibility.

The other major requirement will be for a longer propeller shaft compatible with the increased PSRU case length. This shaft as well as the longer pinion gear "shell" will see increased bending stress due to their increased spans between bearing support locations. This must be accounted for in determining wall thickness and detail of both.

The 0.420:1 ratio PSRU drawing shows a lube oil delivery tube concentric within the pinion gear "shell". The 0.477:1 ratio PSRU drawing does not. An increased length oil delivery tube will be necessary where a lube oil delivery tube is required.

The extension ring may be robust beyond the outline of the PSRU case and may extend upward well beyond the case to receive supports attached to the crankcase, cylinder heads, or even airframe. This should in some measure compensate for the longer lever arm bequeathed to propeller precession loads by the forward extended PSRU assembly. The crankcase/cylinder head support link has precedent in a factory authorized similar scheme as well as unlimited racer practice.

Larger Diameter Gears

Exact specification and drawing details of the Allison PSRU are not in hand, so measured hardware dimensions and assumptions are used for the Allison entries in Table 1, which hint strongly at the blessing available with increased gear center-to-center distance. However, this blessing is not easily secured for the Merlin without major modification.

The Allison PSRU is comprised of sturdy front and rear cases attached to one another via a peripheral and bountiful bolt pattern. This PSRU case assembly, much like the crankcases of many pre-WWII motorcycles, is complete within itself and contains the bearings for both ends of the propeller and input shafts. The rear case is bolted to the engine crankcase with another impressively sturdy peripheral bolt pattern.

An extended Allison PSRU center-to-center distance involves existing shaft and bearing components assembled with their new gears into an enlarged pair of front and rear PSRU cases with a new and larger bolt pattern (if required) joining the cases. The rear case naturally retains the hole pattern that bolts up to the crankcase.

There are apparently three PSRU case sizes and center-to-center choices in existence for the Allison. The largest (10.0") would accommodate a 0.344:1 ratio (21/61) set of gears to the R-R Merlin 25° 4.1088" diametral pitch pattern (though to fit the Allison PSRU exactly the diametral pitch should be altered to 4.1000", thus leading to a slightly stronger tooth). These, in either case, would handle the 3,400 hp @ 3,400 rpm power rating with no more pinion tooth bending stress than is routinely managed by stock R-R 71-tooth, 25°, 0.420:1 ratio gearsets in a race prepared Merlins.

Pitch line velocities with the larger Allison size gears and a race prepared engine run above the 5,000 fpm often regarded as a "turning point" where gear lubrication becomes more of a concern. It is well to be alert to coking, flooding, abnormal oil temperature rise, roller bearing skidding and tooth bluing, all of which are indicative of gear lube failure. All these are curable but could involve some expensive hardware.

Of more than passing interest is that the

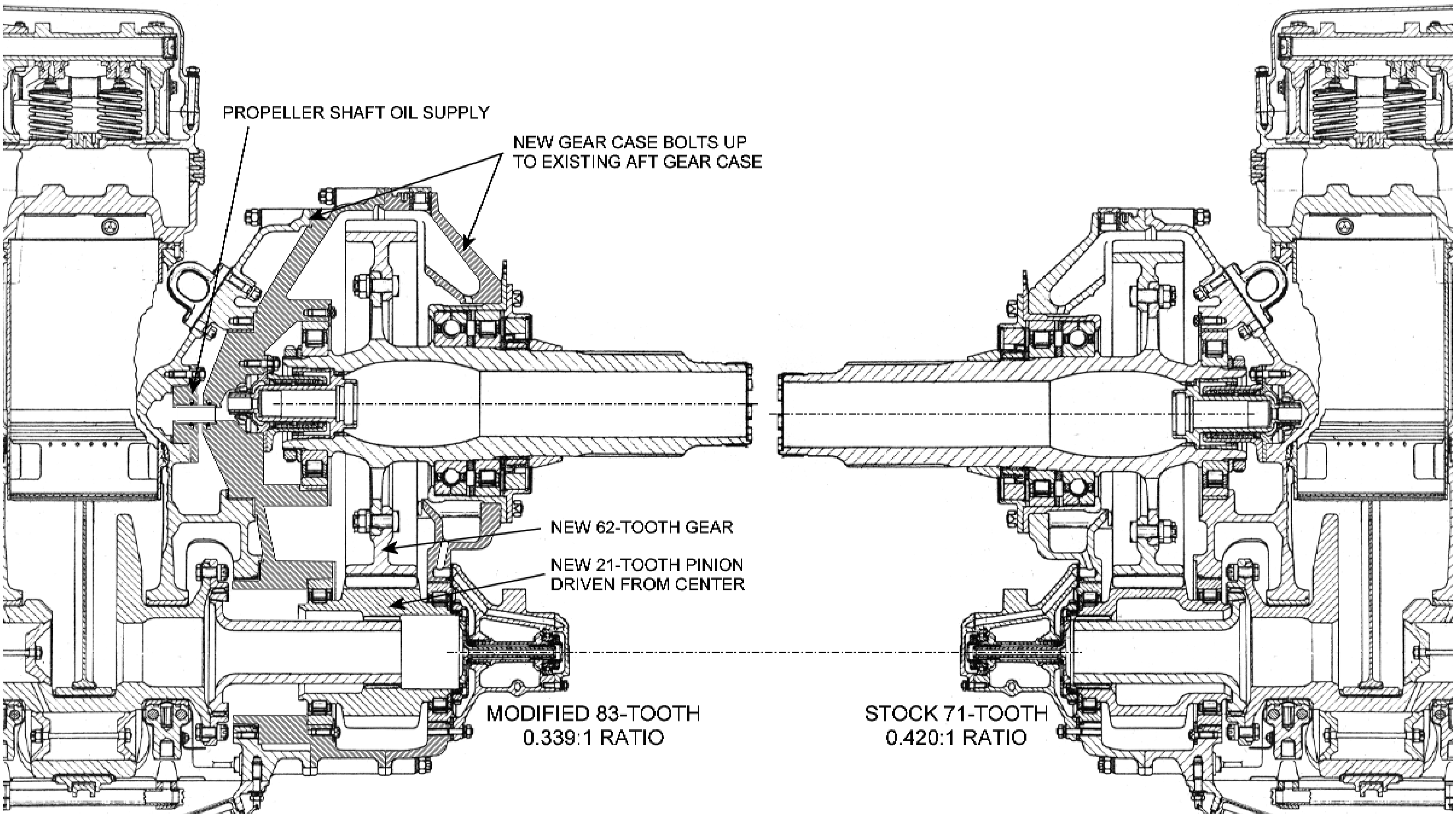


Fig. 5. On the left is a hypothetical 83-tooth 0.339:1 ratio modified Merlin gearset. The original 71-tooth 0.420 ratio Merlin gearset is on the right. A new, larger gear case, shown in bold cross-hatch, has protrusions on its aft half that pilot into existing bearing bores. Accommodations exist to supply oil to the propeller shaft, and the whole assembly bolts up the existing aft gear case half. The quill shaft and propeller shaft are retained. A new 21-tooth pinion is externally identical to the original, but internal splines that mate with the quill shaft splines are moved to the pinion center, reducing the tendency for an end-driven pinion to twist under load. The 62-tooth gear is larger than the 50-tooth part it replaces, resulting in an offset propeller shaft.

Allison pinion is wider than the gear it drives. This may suggest a way to improve Merlin pinion durability as well. It reduces the possibility of a high bending load at the vulnerable unsupported end of the straight cut spur gear tooth. A load applied anywhere along a gear tooth causes local stress and deflection. The same load applied at the abrupt end of a tooth causes higher stress and deflection because there is less material available to resist the load force. Gears are imperfect and don't load one another in a linearly uniform manner. The smaller pinion is most vulnerable to high end-of-tooth stress and deflection. Extending the pinion teeth beyond the point of possible loading renders this unfortunate loading nearly impossible.

This leaves us happy for the Allison folks but securing similar blessing for Merlin drivers promises a formidable task. The rear case portion of the Merlin PSRU is also the front of the crankcase complete with the rear bearing locating and support diameters for both PSRU shafts. The lower PSRU shaft is, as it should be, in line with the Merlin crankshaft. Increasing PSRU gear center to center distance requires that the upper shaft not only be raised but be moved well forward to clear the crankcase portion which formerly served as the smaller center to center PSRU aft case. The pinion must also move forward (fig 5).

This could all be handled with a new PSRU case containing the aft shaft bearing locating and support detail for both shafts as well as the propeller oil feed details for the prop shaft. This new case would also have a complete attach bolt pattern to fit the existing Merlin PSRU aft case. It would extend well beyond this bolt pattern to accommodate the larger gears and include beyond them another bolt pattern for attachment of the forward PSRU case.

The forward PSRU case component would contain the bearing location and support detail for the forward ends of both shafts and accommodate the crankshaft oil feed fittings and other details required at this location. The existing propeller shaft may be useable but the aft end site of the propeller shaft will require oil plumbing. While the existing quill shaft can probably be retained, a new pinion that duplicates the original externally will be required. The new pinion will have the splines that mate with the quill shaft displaced aft so that the pinion is driven from its center rather than its end. Front features of the pinion that interface with the auxiliary drives and provide lubrication will remain unchanged.

Direct support linkage to the engine should be provided from the new PSRU case as with the contemplated increased face width PSRU. It appears as though it is intended that PSRU scavenge oil be dumped directly from the aft section of the PSRU into the front of the oil "pan" and this pathway must be maintained.

The merits of this hypothetical Merlin PSRU concept and the appeal of larger gears running at increased center-to-center distance are made clear by the **R-R 83 @ 0.339 Race** entry in Table 1. This concept initially looks promising and probably warrants further consideration, that task will be left to new dogs with new tricks.

Conclusion

We have viewed the R–R Merlin PSRU gearing through the technical "eyes" of its era and then brought those well proven techniques from amongst layout drafting practices to analytical geometry convenient for today's hand held calculators. This was as far as my "old dog, no new tricks" mindset would go.

Kimble McCutcheon then ushered us into the 21st Century (five years late!) with Excel spreadsheet magic that will do all the hard work and allow investigating "what if" scenarios in microseconds. I hope that the *Torque Meter* articles, sketches and derivation leading up to this point will help readers better understand gearing and its analysis and so aid in the use of the spreadsheet. I would be pleased to hear of any results from this work or of any errors that are discovered and look forward to learning of gearing "breakthroughs" amongst Reno Racers. More background and details, including the Excel spreadsheet used to prepare Table 1 are available on the AEHS web site.

Acknowledgements

The AEHS is grateful to Industrial Press, Inc. for granting permission for use of the formed tooth, rotary cutter and hob cutter gear generation illustrations.

Errata

In Part 2 (Vol. 5, No. 3), page 36, the numerator in the equation for ϕ_P under ANGULAR CONTRIBUTIONS should be r_P, not r_f.

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